PHYS 100C, Lecture #8

* Reflection/transmission at conducting interface

So what happened to EM wave if it can't get through? It gets reflected?

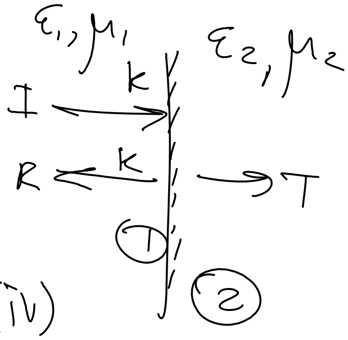
Boundary conditions:

$\varepsilon_1 E_1^{\perp} - \varepsilon_2 E_2^{\perp} = \sigma$ (i)

$B_1^{\perp} - B_2^{\perp} = 0$ (ii)

$E_1^{\parallel} - E_2^{\parallel} = 0$ (iii)

$\frac{B_1^{\parallel}}{\mu_1} - \frac{B_2^{\parallel}}{\mu_2} = \vec{K} \times \hat{n}$ (iv)

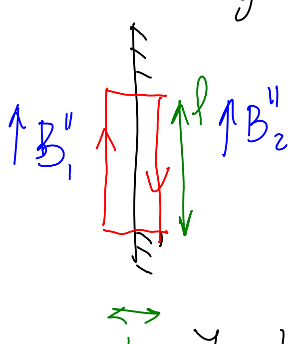


For conductors $\vec{K}$ (surface current per unit length) must be equal to 0, otherwise

J (current per area) $\to \infty$

and $E = \frac{J}{\sigma} \to \infty$ as well

*(We can have finite current densities J, but then total current per unit length can be made $\to 0$ by tightening the Ampere's Loop $dz \to 0$, $|\vec{K}| = J \cdot dz \cdot \ell \to 0$)



Just like in Lectures 2,3:

Incident wave:

$$E_I = \tilde{E}_{0I} e^{i(k_1 z - \omega t)} \; ; \; B_I = \frac{\tilde{E}_{0I}}{v_1} e^{i(k_1 z - \omega t)}$$

Reflected:

$$E_R = \tilde{E}_{0R} \cdot e^{i(-k_1 z - \omega t)} \; ; \; B_R = -\frac{\tilde{E}_{0R}}{v_1} e^{i(-k_1 z - \omega t)}$$

Transmitted wave has the solution derived above:

$$E_T = \tilde{E}_{0T} e^{i(\tilde{k}_2 z - \omega t)} \; ; \; B_R = \frac{\tilde{k}_2 \tilde{E}_{0T}}{\omega} e^{i(\tilde{k}_2 z - \omega t)}$$

Same as before, but $\tilde{k}$ is complex.

Boundary conditions:

(i) & (ii) yield nothing

since $E_\perp = B_\perp = 0$ (ALSO means $\sigma = 0$)

(iii): $\tilde{E}_{0I} + \tilde{E}_{0R} = \tilde{E}_{0T}$

(iv): $\dfrac{\tilde{E}_{0I}}{\mu_1 v_1} - \dfrac{\tilde{E}_{0R}}{\mu_1 v_1} = \dfrac{\tilde{k}_2}{\mu_2 \omega} \tilde{E}_{0T}$

Solutions are:

$$\tilde{E}_{0R} = \left(\frac{1 - \tilde{\beta}}{1 + \tilde{\beta}} \right) \tilde{E}_{0I}$$

$$\tilde{E}_{0T} = \left(\frac{2}{1 + \tilde{\beta}} \right) \tilde{E}_{0I}$$

(Same as before but $\tilde{\beta} = \dfrac{\mu_1 v_1}{\mu_2 \omega} \tilde{k}_2$ is complex now!)

Perfect conductor $\sigma \to \infty$

$$\Rightarrow \tilde{k}_2 \to \infty$$
$$\Rightarrow \beta \to \infty$$

and

$$\tilde{E}_{0R} = -\tilde{E}_{0I} \quad ; \quad \tilde{E}_{0T} = 0$$

$$R \equiv \left(\frac{|\tilde{E}_{0R}|}{|\tilde{E}_{0I}|}\right)^2 = 1 \qquad T = 0$$

100% reflection at normal incidence, reflected wave is 180° out of phase w.r.to incident.

Please verify this every day by looking at the mirror reflection (also check for vampires!)

* PHYSICS SUMMARY:

EM waves in conductors (metals) induce currents (confined to "skin depth" near the surface) which "screen" the wave from passing through conductor.

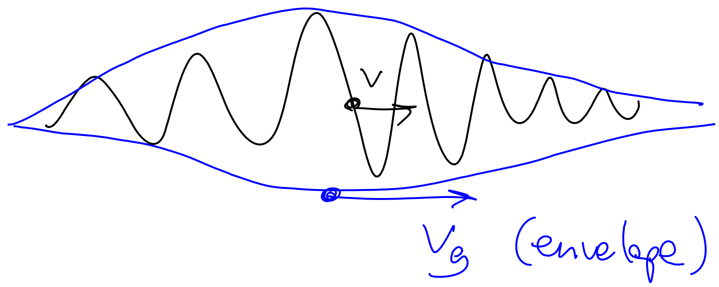
As a result, waves get reflected (primarily), giving rise to shiny objects, mirrors etc.

* Dispersion:

ε ($\Rightarrow n, v$) depends on ω

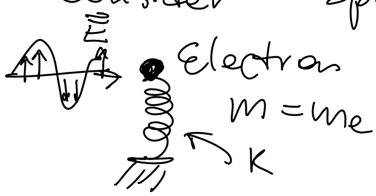
Phase velocity $v = \frac{\omega}{k}$

Group velocity $v_g = \frac{\partial \omega}{\partial k}$



Why does ε (and v, n etc.) depend on ω?

Consider "spring" model of electron:



$$\omega = \sqrt{\frac{K}{m_e}}$$

Eq. of motion:

$$\underbrace{m\frac{\partial^2 x}{\partial t^2}}_{F=ma} + \underbrace{m\gamma\frac{\partial x}{\partial t}}_{\text{dissipation}} + \underbrace{m\omega_0^2 x}_{kx} = \underbrace{eE_0 \cdot \cos\omega t}_{\text{driving force}}$$

General Solution: $x = \tilde{x}_0 \cdot e^{-i\omega t}$

$$-\omega^2 x - i\gamma\omega x + \omega_0^2 x = \frac{eE_0}{m} e^{-i\omega t}$$

$$\tilde{x}_0 = \frac{e/m}{\omega_0^2 - \omega^2 - i\gamma\omega} E_0$$

Polarization $\tilde{p} = e\tilde{x}$

$$\tilde{P} = \sum_{\substack{\text{ALL} \\ \text{ELECTRONS/VOLUME}}} \tilde{p} = \varepsilon_0 \tilde{\chi}_e \tilde{E}$$

$$\tilde{\varepsilon} = \varepsilon_0 (1 + \tilde{\chi}_e)$$

all molecules

$$\tilde{\varepsilon}_R = 1 + \frac{Ne^2}{m\varepsilon_0} \cdot \sum_j \frac{f_j}{\omega_j^2 - \omega^2 - i\gamma\omega}$$

Planar wave $E = E_0 \cdot e^{i(\tilde{k}z - \omega t)}$

where $\tilde{k} = \sqrt{\tilde{\varepsilon}\mu_0}\,\omega = k + i\frac{\alpha}{2}$

$$E = E_0 \cdot e^{-\frac{\alpha z}{2}} \cdot e^{i(kz - \omega t)}$$

α is absorption coefficient

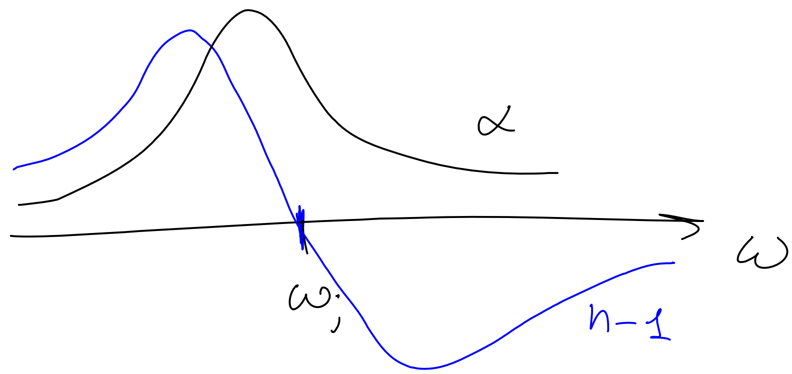
$$n = \frac{ck}{\omega}$$

for small dispersive corrections:

$$\sqrt{1+\varepsilon} \simeq 1 + \varepsilon/2$$

$$\tilde{k} = \frac{\omega}{c}\left[1 + \frac{Ne^2}{2m\varepsilon_0} \sum_j \frac{f_j}{\omega_j^2 - \omega^2 - i\gamma_j\omega}\right]$$

$$n = \frac{ck}{\omega} \cong 1 + \frac{Ne^2}{2m\varepsilon_0} \sum_j \frac{f_j(\omega_j^2 - \omega^2)}{(\omega_j^2 - \omega^2)^2 + \gamma_j^2\omega^2}$$



Far away from resonances,

$$n \sim 1 + \dots \sum \frac{\dots}{\omega_j^2 - \omega^2}$$

ω_j are in UV range

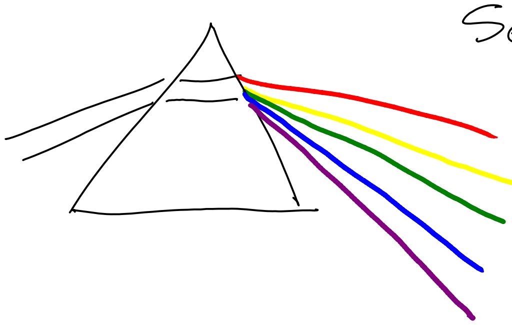
for visible light $\omega < \omega_j$

$$\frac{1}{\omega_j^2 - \omega^2} = \frac{1}{\omega_j^2\left(1 - \omega^2/\omega_j^2\right)} \simeq \frac{1}{\omega_j^2}\left(1 + \frac{\omega^2}{\omega_j^2}\right)$$

$$n = 1 + A\left(1 + B/\lambda^2\right)$$

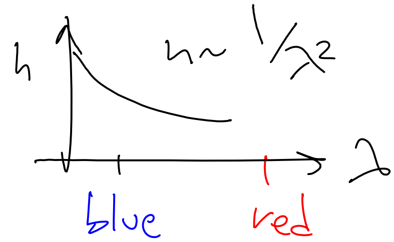
Cauchy's FORMULA

* Physics consequences:



See Cover ART "Dark side of the Moon" by PINK FLOYD

n increases for shorter wavelengths (blue/violet)



$n \sim 1/\lambda^2$

RAINBOWS:

UV light (bad for skin, can damage DNA) gets substantially absorbed by atmosphere (since $\omega_j \approx$ UV)

For very large frequencies

$\omega > \omega_j \qquad n < 1 \quad$ (X-RAYS)