Physics 100C

Monday, March 29, 2010
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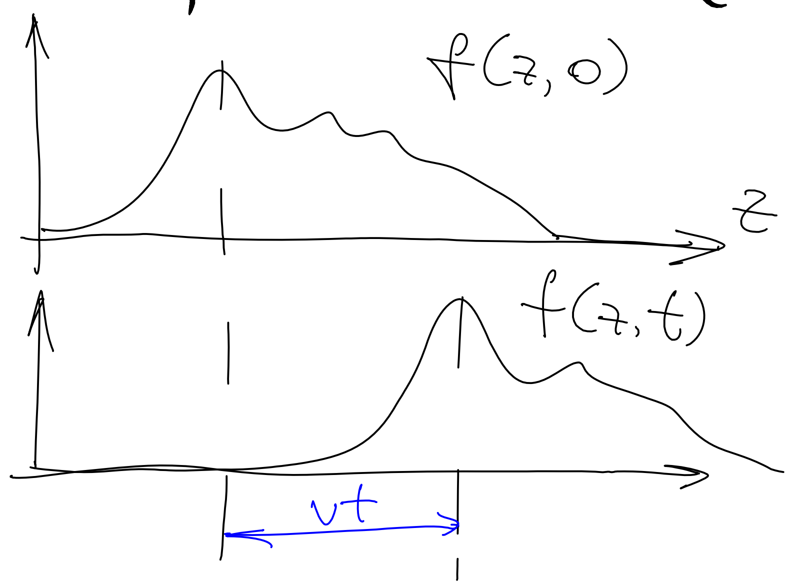
Lecture #1

Waves, Brief Summary (1D)



If $f(z,t)$ can be represented as a function of $u = z - vt$:

$$f(z,t) = g(z - vt)$$

it represents a wave of constant shape, traveling at velocity v (along $\vec{z}$ direction)

For many systems wave equation has differential form:

$$\frac{\partial^2 f}{\partial z^2} = \frac{1}{v^2} \cdot \frac{\partial^2 f}{\partial t^2} \quad \text{(1D version)}$$

See pp. 365-366 for example of string under tension

Allows solutions:

$$f(z,t) = g(u) = g(z-vt)$$

$$\frac{\partial f}{\partial z} = \frac{\partial g}{\partial u} \cdot \underbrace{\frac{\partial u}{\partial z}}_{=1} = \frac{\partial g}{\partial u}$$

$$\frac{\partial f}{\partial t} = \frac{\partial g}{\partial u} \cdot \underbrace{\frac{\partial u}{\partial t}}_{=-v} = -v\frac{\partial g}{\partial u}$$

$$\frac{\partial^2 f}{\partial z^2} = \frac{\partial}{\partial z}\left(\frac{\partial g}{\partial u}\right) = \frac{\partial^2 g}{\partial u^2}$$

$$\frac{\partial^2 f}{\partial t^2} = \frac{\partial}{\partial t}\left(-v\frac{\partial g}{\partial u}\right) = v^2\frac{\partial^2 g}{\partial u^2}$$

So that: $$\frac{\partial^2 g}{\partial u^2} = \frac{1}{v^2} \cdot \frac{\partial^2 f}{\partial t^2}$$

Sinusoidal waves:

$$f(z,t) = A \cdot \cos(k(z-vt)+\delta)$$

$k = \frac{2\pi}{\lambda}$ (so that when $z-vt$ is incremented by $n\lambda$, phase shift is $= 2\pi n$)

"wavenumber"

Also: $$T = \frac{\lambda}{v} = \frac{1}{\nu}$$

$$\omega = \frac{2\pi}{T} = 2\pi\nu = k \cdot v$$

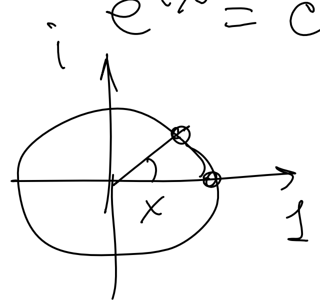
$$f(z,t) = A \cdot \cos(kz - \omega t + \delta)$$

wave traveling to the right (along $\vec{z}$)

For waves traveling left? CHANGE sign of k OR ω.

COMPLEX notation:

$$e^{ix} = \cos x + i \sin x$$

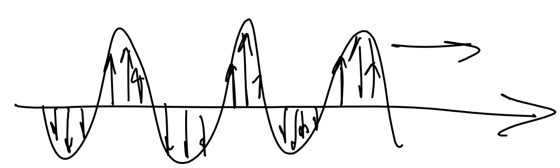


e.g. $e^{i\pi} = -1$

$$f(z,t) = \mathrm{Re}\left[\tilde{A} \cdot e^{i(kz - \omega t)}\right]$$

very useful (see Ex. 9.1)

Longitudinal: 

Transverse: 

EM in vacuum:

Maxwell equations:

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0} = 0 \qquad \text{Gauss}$$

$$\vec{\nabla} \cdot \vec{B} = 0 \qquad \text{No MAGN. MONOPOLES}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \text{FARADAY}$$

$$\vec{\nabla} \times \vec{B} = \underbrace{\cancel{\mu_0 \vec{J}}}_{0} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \qquad \text{Ampere (+MAXWELL)}$$

Coupled 1st ORDER

TO BE CONTINUED...