

PHYS 100C, Homework #3, due Monday, April 26 before class

Problem 9.19

(a) Show that the skin depth in a poor conductor ($\sigma \ll \omega\epsilon$) is $(2/\sigma)\sqrt{\epsilon/\mu}$ (independent of frequency). Find the skin depth (in meters) for (pure) water.

(b) Show that the skin depth in a good conductor ($\sigma \gg \omega\epsilon$) is $\lambda/2\pi$ (where λ is the wavelength *in the conductor*). Find the skin depth (in nanometers) for a typical metal ($\sigma \approx 10^7 (\Omega \text{ m})^{-1}$) in the visible range ($\omega \approx 10^{15}/\text{s}$), assuming $\epsilon \approx \epsilon_0$ and $\mu \approx \mu_0$. Why are metals opaque?

(c) Show that in a good conductor the magnetic field lags the electric field by 45° , and find the ratio of their amplitudes. For a numerical example, use the “typical metal” in part (b).

Problem 9.24 Find the width of the anomalous dispersion region for the case of a single resonance at frequency ω_0 . Assume $\gamma \ll \omega_0$. Show that the index of refraction assumes its maximum and minimum values at points where the absorption coefficient is at half-maximum.

Problem 9.27 Show that the mode TE_{00} cannot occur in a rectangular wave guide. [*Hint:* In this case $\omega/c = k$, so Eqs. 9.180 are indeterminate, and you must go back to 9.179. Show that B_z is a constant, and hence—applying Faraday’s law in integral form to a cross section—that $B_z = 0$, so this would be a TEM mode.]

Problem 9.28 Consider a rectangular wave guide with dimensions $2.28 \text{ cm} \times 1.01 \text{ cm}$. What TE modes will propagate in this wave guide, if the driving frequency is $1.70 \times 10^{10} \text{ Hz}$? Suppose you wanted to excite only *one* TE mode; what range of frequencies could you use? What are the corresponding wavelengths (in open space)?

Problem 9.30 Work out the theory of TM modes for a rectangular wave guide. In particular, find the longitudinal electric field, the cutoff frequencies, and the wave and group velocities. Find the ratio of the lowest TM cutoff frequency to the lowest TE cutoff frequency, for a given wave guide. [*Caution:* What is the lowest TM mode?]