9.21 $R \equiv \left(\frac{E_{0R}}{E_{0I}}\right)^2 = \left|\frac{1-\tilde{\beta}}{1+\tilde{\beta}}\right|^2$

$\tilde{\beta} = \frac{\mu_1 v_1}{\mu_2 \omega}\tilde{k}$ $\quad\quad$ $\tilde{k} = k + ik$

(since $\kappa \simeq k$ for good conductors, see 9.19)

$k \simeq \sqrt{\frac{\sigma\mu\omega}{2}}$ $\cancel{\approx 29}$

$\frac{\mu_1 v_1}{\mu_2 \omega}\cdot k \cong \sqrt{\frac{\sigma\mu_0}{2\omega}}\cdot c \approx 29$

$R = \left|\frac{1-29-i\cdot 29}{1+29+i\cdot 29}\right|^2 = \frac{28^2+29^2}{30^2+29^2} = 0.933$ or $\approx 93\%$

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3:21 PM

9.24

$$n = 1 + A\frac{x}{x^2+\gamma^2\omega^2} \approx 1 + A\frac{x}{x^2+\gamma^2\omega_0^2}$$

where $x = \omega_0^2 - \omega^2$

$$\frac{\partial n}{\partial x} = \frac{A}{x^2+\gamma^2\omega_0^2} - \frac{2x^2A}{(x^2+\gamma^2\omega_0^2)^2} = 0$$

$$\frac{\gamma^2\omega^2 - x^2}{(x^2+\gamma^2\omega^2)^2} = 0$$

$x = \pm\gamma\omega_0$

(we assumed $\omega \approx \omega_0$
or that $\left|\frac{x}{\omega_0^2}\right| = \frac{\gamma}{\omega_0} << 1$) True!

$$\alpha = \frac{A}{x^2+\gamma^2\omega^2} \approx \frac{A}{x^2+\gamma^2\omega_0^2} \quad (\omega \approx \omega_0)$$

$\alpha_{max} = \frac{A}{\gamma^2\omega_0^2}$ when $x=0$

$$\alpha_{1,2}(x = \pm\gamma\omega_0) = \frac{A}{(\pm\gamma\omega_0)^2+\gamma^2\omega_0^2} = \frac{A}{2\gamma^2\omega_0^2}$$

OR $\alpha_{1,2} = \frac{\alpha_{max}}{2}$

9.27 TE_{00} $\quad E_z = 0$

$k = \omega/c$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = i\omega B_z$$

$$-ikE_y = i\omega B_x$$

$$ikE_x = i\omega B_y$$

$$\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} = 0$$

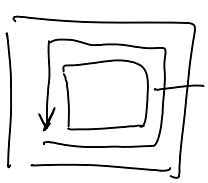
$$\frac{\partial B_z}{\partial y} - ikB_y = -\frac{i\omega}{c^2}E_x$$

$$ikB_x - \frac{\partial B_z}{\partial x} = -\frac{i\omega}{c^2}E_y$$

$$\frac{\partial B_z}{\partial y} = ikB_y - \frac{i\omega}{c^2}\cdot\frac{\omega B_y}{k} = ikB_y - ikB_y = 0$$

$$\frac{\partial B_z}{\partial x} = -ik\cdot\frac{k}{\omega}E_y + \frac{i\omega}{c^2}E_y = 0$$

Since $\frac{\partial B_z}{\partial y} = \frac{\partial B_z}{\partial x} = 0$ for all x and y, $B_z(x,y) = \text{const.}$



Loop

$$\oint E\cdot dl = -A\cdot\int\frac{\partial B_z}{\partial t}$$

$$\frac{\partial B_z}{\partial t} = i\omega B_z = \text{const}$$

Since $E=0$ inside conductor,

$\oint E\cdot dl = 0 \Rightarrow B_z = 0$

9.28 $a = 2.28\,cm$

$b = 1.01\,cm$

$$\nu_{mn} = \frac{\omega_{mn}}{2\pi} = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$\nu_{01} = \frac{c}{2b} = 1.4 \cdot 10^{10}\,Hz \quad < 1.7 \cdot 10^{10}\,Hz = \nu$$

$$\nu_{10} = \frac{c}{2a} = 0.66 \cdot 10^{10}\,Hz < \nu$$

$$\nu_{11} = \frac{c}{2}\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2} = 1.62 \cdot 10^{10}\,Hz < \nu$$

$$\nu_{20} = 2 \cdot \nu_{10} = 1.32 \cdot 10^{10}\,Hz < \nu$$

All others ($\nu_{21}, \nu_{12}, \nu_{30}, \nu_{02}$, etc.) are $> \nu$

9.30 $E_z(x,y) = X(x)\,Y(y)$

$X(x) = A \cdot \sin k_x x + B \cos k_x x$

$E_z = 0$ at $x = 0$, $x = a$

$B = 0$, and

$k_x = \frac{\pi m}{a}$

Similarly for $Y(y)$:

$$E_z = E_0 \cdot \sin\frac{m\pi x}{a} \cdot \sin\frac{n\pi y}{b}$$

$$\frac{\omega_{mn}}{c} = \sqrt{\left(\frac{\pi m}{a}\right)^2 + \left(\frac{\pi n}{b}\right)^2}$$

Lowest TM mode is (11)

$$\omega_{11}^{TM} = c\pi \sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2}$$

Lowest TE mode is (10)

$$\omega_{10}^{TE} = \frac{c\pi}{a}$$

$$\frac{\omega_{11}^{TM}}{\omega_{10}^{TE}} = \sqrt{1 + \left(\frac{a}{b}\right)^2}$$