PHYS 100C Midterm, Thursday May 14, 8AM-9AM (1 hour).

Problem 9.33 Suppose

$$\mathbf{E}(r,\theta,\phi,t) = A\frac{\sin\theta}{r}\left[\cos(kr-\omega t) - (1/kr)\sin(kr-\omega t)\right]\hat{\boldsymbol{\phi}}, \quad \text{with } \frac{\omega}{k} = c.$$

(This is, incidentally, the simplest possible **spherical wave**. For notational convenience, let $(kr - \omega t) \equiv u$ in your calculations.)

(a) Show that **E** obeys all four of Maxwell's equations, in vacuum, and find the associated magnetic field.

(b) Calculate the Poynting vector. Average **S** over a full cycle to get the intensity vector **I**. (Does it point in the expected direction? Does it fall off like r^{-2}, as it should?)

(c) Integrate $\mathbf{I}\cdot d\mathbf{a}$ over a spherical surface to determine the total power radiated. [*Answer:* $4\pi A^2/3\mu_0 c$]

Problem 9.38 Consider the **resonant cavity** produced by closing off the two ends of a rectangular wave guide, at $z = 0$ and at $z = d$, making a perfectly conducting empty box. Show that the resonant frequencies for both TE and TM modes are given by

$$\omega_{lmn} = c\pi\sqrt{(l/d)^2 + (m/a)^2 + (n/b)^2}, \qquad (9.204)$$

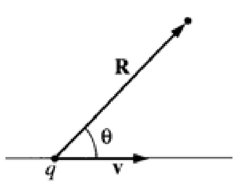
for integers l, m, and n. Find the associated electric and magnetic fields.

Problem 10.14 Show that the scalar potential of a point charge moving with constant velocity (Eq. 10.42) can be written equivalently as

$$V(\mathbf{r},t) = \frac{1}{4\pi\epsilon_0}\frac{q}{R\sqrt{1 - v^2\sin^2\theta/c^2}}, \qquad (10.44)$$

where $\mathbf{R} \equiv \mathbf{r} - \mathbf{v}t$ is the vector from the *present (!)* position of the particle to the field point **r**, and θ is the angle between **R** and **v** (Fig. 10.9). Evidently for nonrelativistic velocities ($v^2 \ll c^2$),

$$V(\mathbf{r},t) = \frac{1}{4\pi\epsilon_0}\frac{q}{R}.$$



Problem 10.19

(a) Use Eq. 10.68 to calculate the electric field a distance d from an infinite straight wire carrying a uniform line charge λ, moving at a constant speed v down the wire.

(b) Use Eq. 10.69 to find the *magnetic* field of this wire.