PHYS 100C, LECTURE # 4

Monday, April 05, 2010

* EM WAVES IN MEDIA:

MAXWELL EQS IN MEDIA

NO FREE CHARGE $\rho = 0$

NO FREE CURRENTS $j = 0$

$$\bar{\nabla} \cdot \bar{D} = 0 \qquad \text{(i)}$$

$$\bar{\nabla} \cdot \bar{B} = 0 \qquad \text{(ii)}$$

$$\bar{\nabla} \times \bar{E} = -\partial B / \partial t \qquad \text{(iii)}$$

$$\bar{\nabla} \times \bar{H} = \partial D / \partial t \qquad \text{(iv)}$$

Linear media $\bar{D} = \varepsilon \bar{E}$

$\bar{H} = \frac{1}{\mu} \bar{B}$

Homogeneous μ, ε indep. of x, y, z

$$\bar{\nabla} \cdot \bar{E} = 0 \qquad \text{(i)}$$

$$\bar{\nabla} \cdot \bar{B} = 0 \qquad \text{(ii)}$$

$$\bar{\nabla} \times \bar{E} = -\frac{\partial B}{\partial t} \qquad \text{(iii)}$$

$$\bar{\nabla} \times \bar{B} = \mu \varepsilon \frac{\partial E}{\partial t} \qquad \text{(iv)}$$

SAME AS MAXWELL EQ'S FOR VACUUM

BUT $\mu_0 \to \mu$

$\varepsilon_0 \to \varepsilon$

Solutions: EM WAVES

with velocity $v = \frac{1}{\sqrt{\varepsilon \mu}} = \frac{c}{n}$

where $n = \sqrt{\frac{\varepsilon \mu}{\varepsilon_0 \mu_0}}$ index of refraction

$\mu \simeq \mu_0$ for most materials

$n \simeq \sqrt{\frac{\varepsilon}{\varepsilon_0}} = \sqrt{\varepsilon_R} \leftarrow$ dielectric constant

$\varepsilon_R = \frac{\varepsilon}{\varepsilon_0} > 1$ (almost always)

$n > 1$ and $v < c$

Similarly, energy density:

$$u = \frac{1}{2}\left(\varepsilon E^2 + \frac{1}{\mu} B^2\right)$$

$$\vec{B} = \frac{1}{v} \hat{k} \times \hat{E} \quad \text{(transverse field)}$$

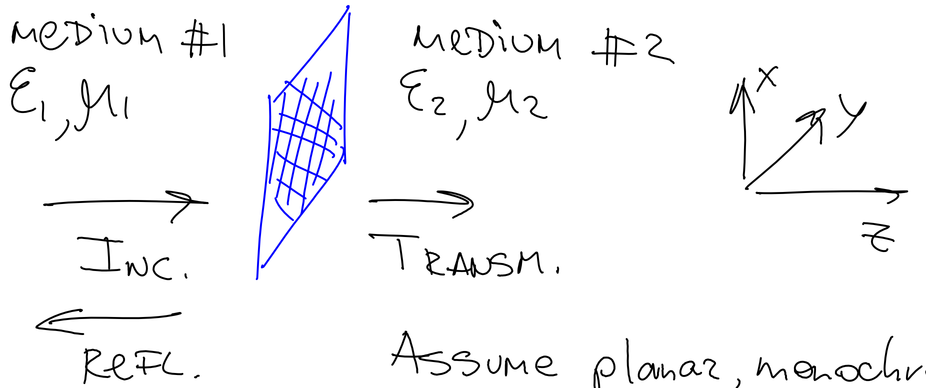
$$\Downarrow$$

$$u = \varepsilon E^2$$

$$\bar{S} = \frac{1}{\mu}(\vec{E} \times \vec{B}) = v \cdot u \cdot \hat{k}$$

$$I = |\langle \bar{S} \rangle| = \frac{1}{2} \varepsilon v E_0^2$$

* Reflection/Transmission at NORMAL INCIDENCE:



Assume planar, monochrom. waves

* BOUNDARY CONDITIONS:

FROM MAXWELL EQ. (i)–(iv):

$$D_1^{\perp} = D_2^{\perp} \quad \text{OR} \quad \varepsilon_1 E_1^{\perp} = \varepsilon_2 E_2^{\perp} \quad \text{(i)}$$

$$B_1^{\perp} = B_2^{\perp} \quad \text{(ii)}$$

$$\bar{E}_1^{\parallel} = \bar{E}_2^{\parallel} \quad \text{(iii)}$$

$$\bar{H}_1^{\parallel} = \bar{H}_2^{\parallel} \quad \text{OR} \quad \frac{B_1^{\parallel}}{\mu_1} = \frac{B_2^{\parallel}}{\mu_2} \quad \text{(iv)}$$

* FOR NORMAL INCIDENCE,

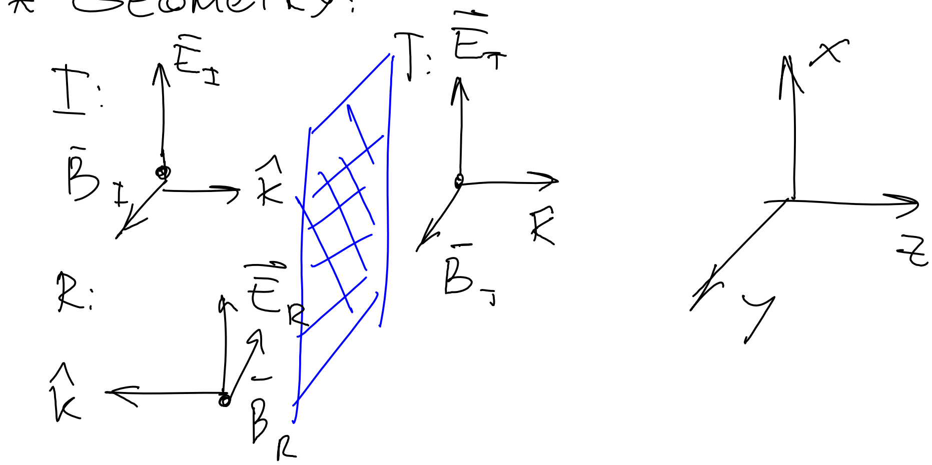
<u>INCIDENT</u> (I) WAVE:

$$\vec{E}_I(z,t) = E_{0I}\, e^{i(kz-\omega t)}$$

$$\vec{B}_I(z,t) = \frac{E_{0I}}{v_1}\, e^{i(kz-\omega t)}\, \hat{y}$$

Where $v_1 = \dfrac{1}{\sqrt{\mu_1 \varepsilon_1}}$

* GEOMETRY:



* Reflected Wave: (R)

$$\vec{E}_R(z,t) = E_{0R}\, e^{i(-k_1 z - \omega t)}\,\hat{x}$$

$$\vec{B}_R(z,t) = -\frac{E_{0R}}{v_1}\, e^{i(-k_1 z - \omega t)}\,\hat{y}$$

Note "−" sign for k_1 in exp. as well as for $\vec{B}_R$ direction.

* Transmitted Wave: (T)

$$\vec{E}_T(z,t) = E_{0T}\, e^{i(k_2 z - \omega t)}\,\hat{x}$$

$$\vec{B}_T(z,t) = \frac{E_{0T}}{v_2}\, e^{i(k_2 z - \omega t)}\,\hat{y}$$

$\omega = k_1 v_1 = k_2 v_2$ = const across boundary

BOUNDARY CONDITIONS
FOR $z=0$

$$E_{0I} + E_{0R} = E_{0T} \qquad (i)$$

$$\frac{1}{\mu_1}\left(\frac{1}{v_1} E_{0I} - \frac{1}{v_1} E_{0R}\right) = \frac{1}{\mu_2}\left(\frac{1}{v_2} E_{0T}\right) \qquad (ii)$$

(iii) and (iv) are not useful
since $E^{\parallel} = B^{\parallel} = 0$ (NORMAL INCIDENCE)

(ii) REWRITTEN:

$$E_{0I} - E_{0R} = \frac{\mu_1 v_1}{\mu_2 v_2} E_{0T} \qquad (ii)$$