PHYS 100C

Wednesday, March 31, 2010

LECTURE #2:

EM in vacuum: (CONT'D)

Maxwell equations:

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0} = 0 \qquad \text{Gauss}$$

$$\vec{\nabla}\cdot\vec{B} = 0 \qquad \text{NO MAGN. MONOPOLES}$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \qquad \text{FARADAY}$$

$$\vec{\nabla}\times\vec{B} = \underbrace{\cancel{\mu_0\vec{J}}}_{0} + \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t} \qquad \text{Ampere (+MAXWELL)}$$

Coupled 1st ORDER

CURL IT AWAY:

$$\nabla\times(\nabla\times E) = \nabla\underbrace{(\cancel{\nabla\cdot E})}_{0} - \nabla^2 E = -\nabla^2 E =$$

$$= \nabla\times\left(-\frac{\partial B}{\partial t}\right) = -\frac{\partial}{\partial t}(\nabla\times B) = -\mu_0\epsilon_0\frac{\partial^2 E}{\partial t^2}$$

$$\nabla\times(\nabla\times B) = \nabla(\cancel{\nabla\cdot B})^{0} - \nabla^2 B = -\nabla^2 B =$$

$$= \nabla\times\left(\mu_0\varepsilon_0\frac{\partial E}{\partial t}\right) = \mu_0\varepsilon_0\frac{\partial}{\partial t}(\nabla\times E) =$$

$$= -\mu_0\varepsilon_0\cdot\frac{\partial^2 B}{\partial t^2}$$

Therefore:

$$\boxed{\begin{aligned} \nabla^2 E &= \mu_0\varepsilon_0\frac{\partial^2 E}{\partial t^2} \\ \nabla^2 B &= \mu_0\varepsilon_0\cdot\frac{\partial^2 B}{\partial t^2} \end{aligned}}$$

3D wave equations!

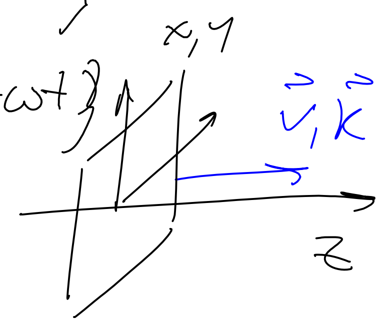
with velocity $\boxed{c^2 = \frac{1}{\mu_0\varepsilon_0}}$ SPEED OF LIGHT IN VACUUM $3\cdot 10^8$ m/s

Monochromatic Plane Waves

Waves traveling along z,
have NO x-y dependence:

$$\vec{E}(z,t) = \mathrm{Re}\left\{\vec{E}_0 \cdot e^{i(kz-\omega t)}\right\}$$

$$\vec{B}(z,t) = \mathrm{Re}\left\{B_0 \cdot e^{i(kz-\omega t)}\right\}$$



Gauss Law:

$$\vec{\nabla}\cdot\vec{E} = 0$$

$$\underbrace{\frac{\partial E_x}{\partial x}}_{=0} + \underbrace{\frac{\partial E_y}{\partial y}}_{=0} + \frac{\partial E_z}{\partial z} = 0$$

(because no x-y dependence)

$$\frac{\partial E_z}{\partial z} = ik E_0^z = 0 \Rightarrow E_0^z = 0$$

Similarly $B_0^z = 0$

EM waves are <u>transverse</u>!

$\vec{E}$ & $\vec{B}$ vectors $\perp$ propagation

Faraday Law:

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & 0 \end{vmatrix} = -\frac{\partial \vec{B}}{\partial t}$$

$$\Downarrow$$

Along $\hat{x}$:

$$-\hat{x}\frac{\partial}{\partial z}E_y = -\hat{x}\cdot\frac{\partial B_x}{\partial t}$$

$$ik\,E_0^y = -i\omega B_0^x \Rightarrow B_0^x = -\frac{k}{\omega}E_0^y$$

Along $\hat{y}$:

$$\hat{y}\frac{\partial}{\partial z}E_x = -\hat{y}\cdot\frac{\partial B_y}{\partial t}$$

$$ik\,E_0^x = i\omega B_0^y \Rightarrow B_0^y = \frac{k}{\omega}E_0^x$$

If we pick $\hat{x}$ so that its

along B_0; i.e. $B_0^y = 0$

then $E_0^x = 0$ as well

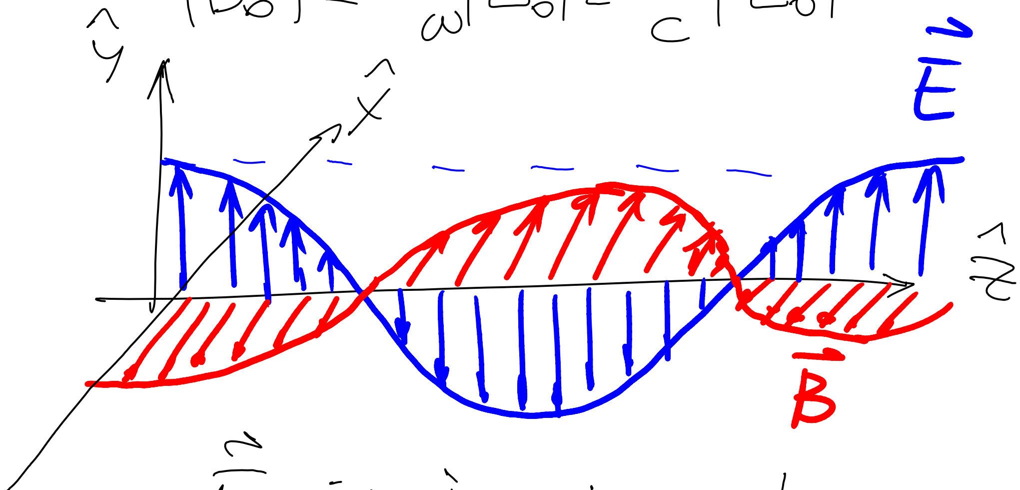
and $\vec{E}_0 \parallel \hat{y}$, $\vec{B}_0 \parallel \hat{x}$,

$$\vec{E}_0 \perp \vec{B}_0$$

OR: $$\vec{B}_0 = \frac{k}{\omega}\left(\hat{z} \times \vec{E}_0\right)$$

$\vec{E}_0$ and $\vec{B}_0$ in phase, and perpendicular to each other

$$|\vec{B}_0| = \frac{k}{\omega}|\vec{E}_0| = \frac{1}{c}|\vec{E}_0|$$



$\vec{E}$ is in xz plane
$\vec{B}$ is in yz plane

* What about $\hat{z}$ component?

$$\hat{z}\frac{\partial}{\partial x}E_y - \hat{z}\frac{\partial}{\partial y}E_x = -\frac{\partial B_z}{\partial t}\hat{z}$$

$$B_z = 0 \; ; \quad \frac{\partial E_y}{\partial x} = \frac{\partial E_x}{\partial y} = 0$$

because waves are PLANAR

(no x-y dependence)

* Also: Ampere-Maxwell Eq. does not produce any new info
Solution above already satisfies.