PHYS 100C, Lecture 14

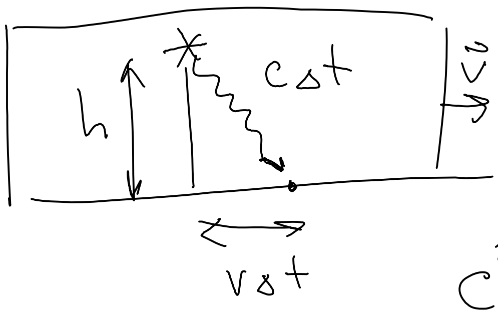
Tuesday, May 19, 2009
8:00 PM

Einstein postulates:

1. Laws of physics are frame of reference-invariant
2. Speed of light is frame-invariant

Many consequences:

Time dilation:



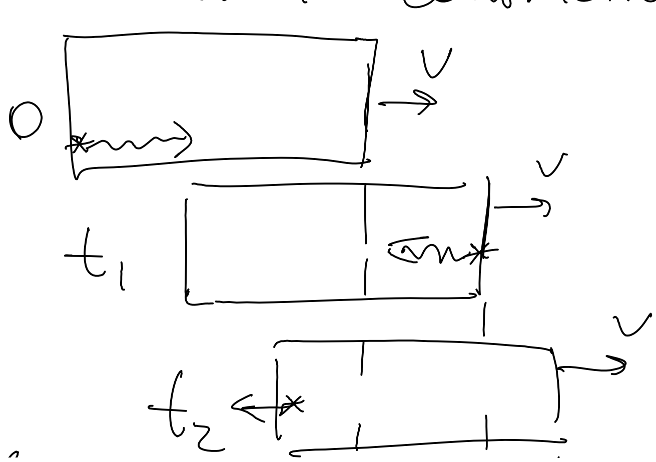
$$h = c\Delta t'$$

$$h^2 = c^2\Delta t^2 - v^2\Delta t^2$$

$$c^2\Delta t'^2 = c^2\Delta t^2 - v^2\Delta t^2$$

$$\Delta t = \frac{\Delta t'}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma\Delta t'$$

Lorentz contraction:

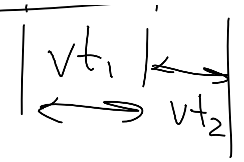


$$ct_1 - vt_1 = L$$

$$ct_2 + vt_2 = L$$

$$t_1 = \frac{L}{c-v}$$

(time to travel back)



$$t_2 = \frac{L}{c+v}$$

$$t = t_1 + t_2 = L\left(\frac{1}{c-v} + \frac{1}{c+v}\right) =$$

$$= L \frac{2c}{c^2 - v^2} = \frac{2L}{c} \cdot \gamma^2$$

In train frame:

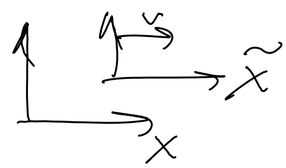
$$\tilde{t} = \frac{2\tilde{L}}{c}$$, but since

$$t = \tilde{t} \cdot \gamma \Rightarrow \frac{2L}{c} \cdot \gamma^2 = \frac{2\tilde{L}}{c} \cdot \gamma$$

$$\tilde{L} = L \cdot \gamma$$

General Lorentz transforms:

$$\tilde{x} = \gamma(x - vt)$$



(replaces $\tilde{x} = x - vt$)

(Lorentz contraction)

Then by equivalence

$$x = \gamma(\tilde{x} + v\tilde{t})$$

OR, plug in $\tilde{x}$ expression:

$$x = \gamma(\gamma(x - vt) + v\tilde{t}) = \gamma^2 x - \gamma^2 vt + \gamma v\tilde{t}$$

$$x = \gamma(\gamma(x - vt) + v\tilde{t}) = \gamma^2 x - \gamma^2 vt + \gamma v\tilde{t}$$

$$x(1-\gamma^2) = \gamma v(\tilde{t} - \gamma t)$$

$$1-\gamma^2 = 1 - \frac{1}{1-v^2/c^2} = \frac{1-v^2/c^2 - 1}{1-v^2/c} = \frac{-v^2/c^2}{1-v^2/c^2}$$

$$-x\cdot\gamma^2\cdot\frac{v^2}{c^2} = \gamma v(\tilde{t} - \gamma t)$$

$$-\gamma\cdot x\cdot\frac{v}{c^2} = \tilde{t} - \gamma t$$

$$\tilde{t} = \gamma\left(t - \frac{v\cdot x}{c^2}\right)$$

So:

$$\tilde{x} = \gamma(x - vt)$$

$$\left.\begin{aligned}\tilde{y} &= y\\ \tilde{z} &= z\end{aligned}\right\} \text{no contraction} \perp \vec{v}$$

$$\tilde{t} = \gamma\left(t - \frac{vx}{c^2}\right)$$

For an object moving with velocity $u = \frac{\partial x}{\partial t}$:

$$\partial\tilde{x} = \gamma(\partial x - v\,\partial t)$$

$$\partial\tilde{t} = \gamma\left(\partial t - \frac{v\,\partial x}{c^2}\right)$$

$$\tilde{u} = \frac{\partial\tilde{x}}{\partial\tilde{t}} = \frac{\gamma(\partial x - v\partial t)}{\gamma\left(\partial t - \frac{v\partial x}{c^2}\right)}$$ divide by ∂t:

$$\tilde{u} = \frac{\frac{\partial x}{\partial t} - v}{1 - \frac{v \cdot \partial x}{c^2 \cdot \partial t}}$$ $\frac{\partial x}{\partial t} \equiv u$

$$\tilde{u} = \frac{u - v}{1 - \frac{v \cdot u}{c^2}}$$

Introduce Four-vector:

$x^0 \equiv ct$ $\quad\quad$ $\tilde{x}^0 = \gamma(x^0 - \beta x^1)$

$x^1 = x$ $\quad\quad$ $\tilde{x}^1 = \gamma(x^1 - \beta x^0)$

$x^2 = y$ $\quad\quad$ $\tilde{x}^2 = x^2$

$x^3 = z$ $\quad\quad$ $\tilde{x}^3 = x^3$

OR, in matrix form:

$$\begin{pmatrix} \tilde{x}^0 \\ \tilde{x}^1 \\ \tilde{x}^2 \\ \tilde{x}^3 \end{pmatrix} = \underbrace{\begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}}_{\Lambda} \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix}$$

where

$$\tilde{X} = \underset{\sim}{\Lambda} \cdot X$$

X is 4-vector Λ is transform

X is 4-vector, Λ is transform matrix. Scalar product:

$\bar{A} \cdot \bar{B} = A_x \cdot B_x + A_y B_y + A_z \cdot B_z = \text{const}$
(frame invariant)

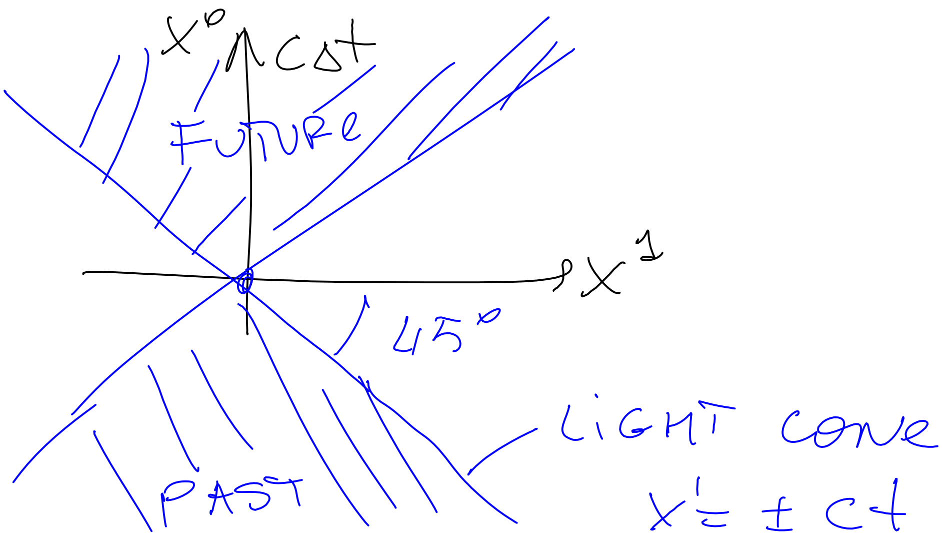
Two events, separated by time Δt and distance d (in some frame)

$I \equiv -c^2 \Delta t^2 + d^2 = \text{const}$
(frame-invariant)

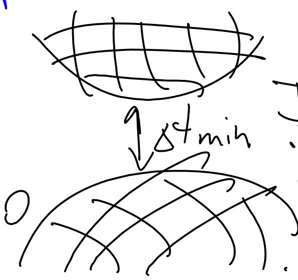
$I < 0$ timelike
(Δt large, d small)

$I > 0$ spacelike

$I = 0$ light like



3-D "hypercone"



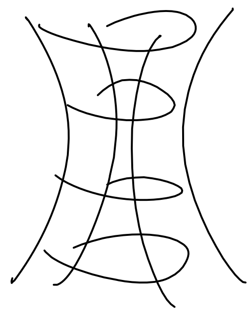
$I < 0$

$\uparrow \Delta t_{min}$

$I = const$ isosurfaces

Δt can't be $= 0$

$$\Delta t_{min} = \frac{|I|}{c}$$



Isosurf. $I = const$

$I > 0$

sign of Δt can be reversed

(events A & B can be made simultaneous, or $t_A > t_B$, or $t_B > t_A$, depending on frame).