Problem 9.2

Monday, April 19, 2010
7:09 PM

$$\frac{\partial f}{\partial z} = A k \cdot \cos(kz) \cdot \cos(kv \cdot t)$$

$$\frac{\partial^2 f}{\partial z^2} = -A k^2 \cdot \sin(kz) \cdot \cos(kvt)$$

$$\frac{\partial f}{\partial t} = -A \cdot k \cdot v \cdot \sin(kz) \cdot \sin(kvt)$$

$$\frac{\partial^2 f}{\partial t^2} = -A k^2 v^2 \cdot \sin(kz) \cdot \cos(kvt) =$$

$$= v^2 \cdot \frac{\partial^2 f}{\partial z^2}$$

* RECALL:

$$\sin X \cdot \cos Y = \frac{1}{2}\left[\sin(X+Y) + \sin(X-Y)\right]$$

then:

$$f = \frac{A}{2}\left[\sin(k(z+vt)) + \sin(k(z-vt))\right]$$

Problem 9.3

Wednesday, April 01, 2009
9:32 PM

$$A_3 e^{i\delta_3} = A_1 e^{i\delta_1} + A_2 e^{i\delta_2}$$

Multiply both sides by complex conjugates (since $|\tilde{a}|^2 = \tilde{a} \cdot \tilde{a}^*$)

$$A_3^2 = \left(A_1 e^{i\delta_1} + A_2 e^{i\delta_2}\right)\left(A_1 \cdot e^{-i\delta_1} + A_2 e^{-i\delta_2}\right)$$

$$A_3^2 = A_1^2 + A_2^2 + A_1 A_2 \left(e^{i(\delta_2 - \delta_1)} + e^{i(\delta_1 - \delta_2)}\right)$$

Since $e^{ix} = \cos x + i \sin x$

$$e^{ix} + e^{-ix} = 2\cos x$$

$$\boxed{A_3^2 = A_1^2 + A_2^2 + 2 A_1 \cdot A_2 \cdot \cos(\delta_2 - \delta_1)}$$

Equating Re parts:

$$A_3 \cdot \cos \delta_3 = A_1 \cdot \cos \delta_1 + A_2 \cdot \cos \delta_2 \qquad (1)$$

Im. parts:

$$A_3 \cdot \sin \delta_3 = A_1 \sin \delta_1 + A_2 \cdot \sin \delta_2 \qquad (2)$$

Divide (2) by (1) (A_3 drops out):

$$\boxed{\tan \delta_3 = \frac{A_1 \cdot \sin \delta_1 + A_2 \cdot \sin \delta_2}{A_1 \cdot \cos \delta_1 + A_2 \cdot \cos \delta_2}}$$

* **Answer:**

$$A_3 = \sqrt{A_1^2 + A_2^2 + 2 A_1 \cdot A_2 \cdot \cos(\delta_2 - \delta_1)}$$

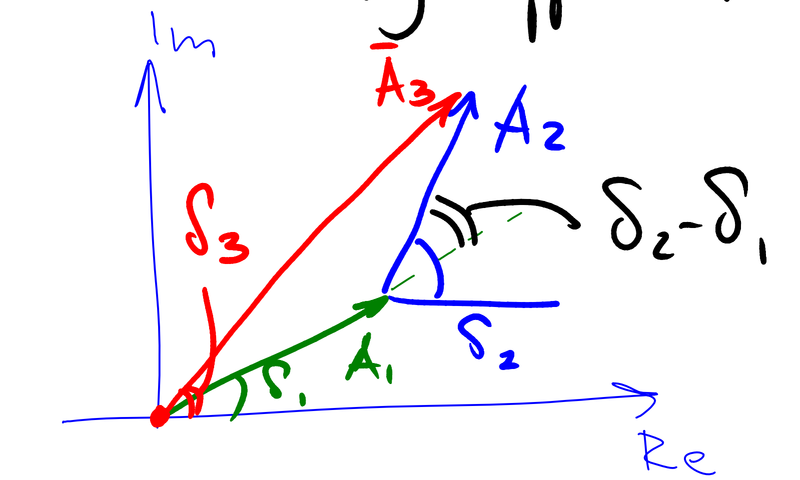
$$\delta_3 = \arctan\left(\frac{A_1 \cdot \sin \delta_1 + A_2 \cdot \sin \delta_2}{A_1 \cdot \cos \delta_1 + A_2 \cdot \cos \delta_2}\right)$$

Problem 9.3, cont'd:

Monday, April 19, 2010
4:04 PM

(Alternative solution)

+ Geometry approach:



From A_1, A_2, A_3 triangle:

$$A_3^2 = A_1^2 + A_2^2 + 2A_1 \cdot A_2 \cos(\delta_2 - \delta_1)$$

$$\tan \delta_3 = \frac{A_2 \cdot \sin \delta_2 + A_1 \cdot \sin \delta_1}{A_2 \cdot \cos \delta_2 + A_1 \cdot \cos \delta_2}$$

(Projections of A_1, A_2 on Re and im axis).

Problem 9.5

Monday, April 19, 2010
4:11 PM

Boundary Conditions at $z=0$:

$$g_I(-v_1t) + h_R(v_1t) = g_T(-v_2t) \qquad (1)$$

$$\frac{\partial g_I}{\partial z}(-v_1t) + \frac{\partial h_R}{\partial z}(v_1t) = \frac{\partial g_T}{\partial z}(-v_2t) \qquad (2)$$

Need to convert $\frac{\partial}{\partial z}$ to $\frac{\partial}{\partial t}$
so we can integrate over t
(cannot integrate over z since $z=0$).

Note that $\frac{\partial g}{\partial z} = \frac{\partial g}{\partial u} \cdot \frac{\partial u}{\partial z}$

and $\frac{\partial g}{\partial t} = \frac{\partial g}{\partial u} \cdot \frac{\partial u}{\partial t}$

where g is g_I, h_R or g_T
and u is $z-v_1t$; $z+v_1t$ or $z-v_2t$
respectively
therefore $\frac{\partial g}{\partial z} = \frac{\partial g}{\partial t} \cdot \frac{\partial u}{\partial z} \cdot \left(\frac{\partial u}{\partial t}\right)^{-1}$

$\frac{\partial u}{\partial z} = 1$ for all functions

$\frac{\partial u}{\partial t}$ is $-v_1$; $+v_1$ and $-v_2$

therefore Eq. (2) becomes:

$$\frac{\partial g_I(-v_1t)}{\partial t} \cdot \left(-\frac{1}{v_1}\right) + \frac{\partial h_R(v_1t)}{\partial t} \cdot \frac{1}{v_1} = \frac{\partial g_T(-v_2t)}{\partial t} \cdot \left(-\frac{1}{v_2}\right) \qquad (3)$$

Multiply $\times(-v_1)$, and integrating $\int dt$:

$$g_I(-v_1t) - h_R(v_1 t) = \frac{v_1}{v_2} g_T(-v_2t) + \text{const.} \qquad (4)$$

Add (1)+(4) $\Rightarrow$ h_R drops out:

$$g_T(-v_2 t) = \frac{2v_2}{v_1+v_2}\, g_I(-v_1 t) + \text{const.}$$

OR, since g_T is a function of $z - v_2 t$

$$\boxed{g_T(z - v_2 t) = \frac{2V_2}{V_1+V_2}\, g_I\!\left(\frac{V_1}{V_2} z - v_1 t\right) + C}$$

(Generally,

$$g_T(x) = \frac{2V_2}{V_1+V_2}\, g_I\!\left(\frac{V_1}{V_2} x\right) + \text{const.}$$
)

h_R can be calculated by adding
$(1) - \frac{V_2}{V_1}\cdot(4)$:

$$g_I(-v_1 t)\cdot\frac{V_1 - V_2}{V_1} + h_R(v_1 t)\cdot\frac{V_1+V_2}{V_1} = 0$$

$$h_R(v_1 t) = \frac{V_2 - V_1}{V_2+V_1}\cdot g_I(-v_1 t) + \text{const.}$$

$$h_R(x) = \frac{V_2 - V_1}{V_2+V_1}\cdot g_I(-x) + \text{const.}$$

OR, as $h_R(z + v_1 t)$:

$$\boxed{h_R(z + v_1 t) = \frac{V_2 - V_1}{V_2+V_1}\cdot g_I(-z - v_1 t) + C}$$

C is the same constant (e.g. (1) for $t=0$)

Problem 9.10

Monday, April 19, 2010
5:01 PM

$$I = 1{,}300\ \frac{W}{m^2} = \langle S \rangle = \frac{1}{2}\varepsilon_0 c E_0^2$$

$$P_1 = \frac{1}{2}\varepsilon_0 E_0^2 = \frac{I}{c} = \frac{1300\frac{W}{m^2}}{3\cdot 10^8 m/c} = 4.33\cdot 10^{-6}\ Pa$$

Perfect absorber

FOR PERFECT REFLECTOR

$$P_2 = 2\cdot 4.33\cdot 10^{-6}\ Pa = 8.66\cdot 10^{-6}\ Pa$$

$$P_{ATM} = 10^5\ Pa$$

$$\frac{P_1}{P_{ATM}} = 4.33\cdot 10^{-11}$$

$$\frac{P_2}{P_{ATM}} = 8.66\cdot 10^{-11}$$

Problem 9.11

Monday, April 19, 2010
6:58 PM

$$\langle fg \rangle = \frac{1}{T}\int_0^T a\cdot\cos(\bar{k}\cdot\bar{r} - \omega t + \delta_A)\times$$

$$\times\, b\cdot\cos(\bar{k}\cdot\bar{r} - \omega t + \delta_B)\cdot dt$$

$$\cos X\cdot\cos Y = \frac{1}{2}\left(\cos(X+Y) + \cos(X-Y)\right)$$

$$\langle fg \rangle = \frac{ab}{2T}\int_0^T \cos(2\bar{k}\cdot\bar{r} - 2\omega t + \delta_A + \delta_B)\,dt +$$

$$+\,\frac{ab}{2T}\int_0^T \cos(\delta_A - \delta_B)\cdot dt = \frac{ab}{2}\cos(\delta_A - \delta_B)$$

First term is integral over fluctuating function $\frac{1}{T}\int \cos(\ldots\omega t) \to 0$

Second term is constant over time

Complex notation:

$$\tilde{f} = a\cdot e^{i(k\cdot r - \omega t + \varphi_A)}$$

$$\tilde{g} = b\cdot e^{i(k\cdot r - \omega t + \varphi_B)}$$

$$\frac{1}{2}\tilde{f}\cdot\tilde{g}^* = \frac{1}{2}ab\, e^{i(\varphi_A - \varphi_B)}$$

$$\mathrm{Re}\left\{\frac{1}{2}\tilde{f}\cdot\tilde{g}^*\right\} = \frac{ab}{2}\cdot\cos(\varphi_A - \varphi_B) =$$

$$= \langle fg \rangle$$