

PHYS 100C, LECTURES 21-22

Wednesday, May 19, 2010
8:00 PM

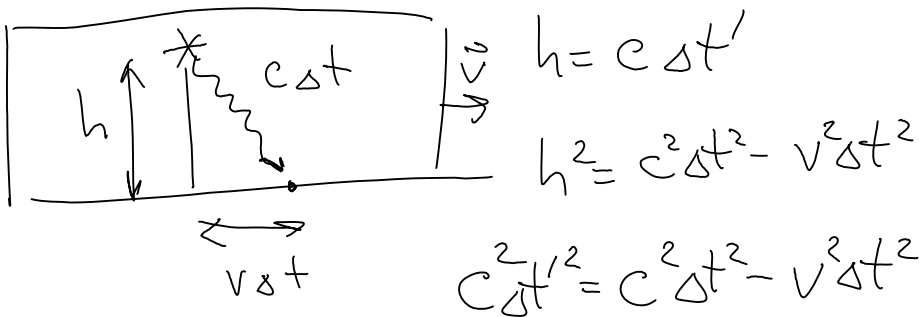
5/17, 5/19

Einstein postulates:

1. Laws of physics are frame-invariant
2. Speed of light is frame-invariant

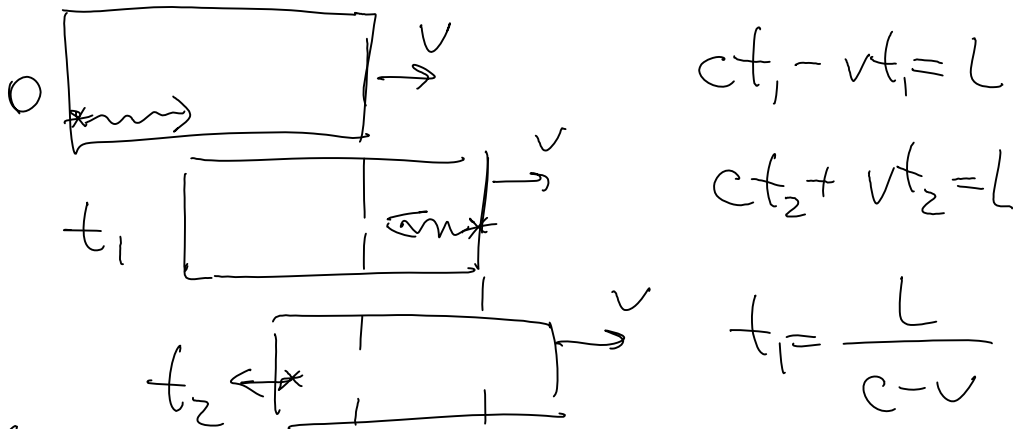
Many consequences:

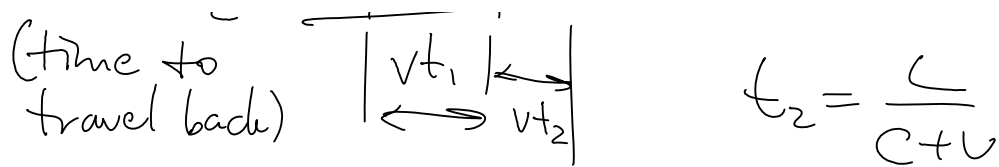
Time dilation:



$$\Delta t = \frac{\Delta t'}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma \Delta t'$$

Lorentz contraction:



(time to travel back)  $t_2 = \frac{L}{c+v}$

$$t = t_1 + t_2 = L \left(\frac{1}{c-v} + \frac{1}{c+v} \right) =$$

$$= L \frac{2c}{c^2 - v^2} = \frac{2L}{c} \cdot \gamma^2$$

In train frame:

$$\tilde{t} = \frac{2\tilde{L}}{c}, \text{ but since}$$

$$t = \tilde{t} \cdot \gamma \Rightarrow \frac{2L}{c} \cdot \gamma^2 = \frac{2\tilde{L}}{c} \cdot \gamma$$

$$\tilde{L} = L \cdot \gamma$$

General Lorentz transforms:

$$\tilde{x} = \gamma(x - vt)$$



(replaces $\tilde{x} = x - vt$)

(Lorentz contraction)

Then by equivalence

$$x = \gamma(\tilde{x} + v\tilde{t})$$

OR, plug in \tilde{x} expression:

$$x = \gamma(x - vt + v\tilde{t}) = \gamma^2 x - \gamma^2 vt + \gamma v\tilde{t}$$

$$x = \gamma(x - vt) + v\tilde{t} = \gamma^2 x - \gamma^2 vt + \gamma v\tilde{t}$$

$$x(1 - \gamma^2) = \gamma v(\tilde{t} - \gamma t)$$

$$1 - \gamma^2 = 1 - \frac{1}{1 - v^2/c^2} = \frac{1 - v^2/c^2 - 1}{1 - v^2/c^2} = \frac{-v^2/c^2}{1 - v^2/c^2}$$

$$-x \cdot \gamma \frac{v^2}{c^2} = \gamma v(\tilde{t} - \gamma t)$$

$$- \gamma \cdot x \cdot \frac{v}{c^2} = \tilde{t} - \gamma t$$

$$\tilde{t} = \gamma \left(t - \frac{v \cdot x}{c^2} \right)$$

So:

$$\tilde{x} = \gamma(x - vt)$$

$$\left. \begin{array}{l} \tilde{y} = y \\ \tilde{z} = z \end{array} \right\} \text{no contraction } \perp \vec{v}$$

$$\tilde{t} = \gamma \left(t - \frac{vx}{c^2} \right)$$

For an object moving with velocity $u = \frac{\partial x}{\partial x_0}$:

$$\partial \tilde{x} = \gamma(\partial x - v \partial t)$$

$$\partial \tilde{t} = \gamma \left(\partial t - \frac{v \partial x}{c^2} \right)$$

$$\tilde{u} = \frac{\partial \tilde{x}}{\partial x} = \frac{\gamma(\partial x - v \partial t)}{\gamma(\partial t - \frac{v \partial x}{c^2})} \quad \text{divide by } \partial t:$$

$$\tilde{u} = \frac{\frac{\partial x}{\partial t} - v}{1 - \frac{v \cdot \partial x}{c^2 \partial t}} \quad \frac{\partial x}{\partial t} \equiv u$$

$$\tilde{u} = \frac{u - v}{1 - \frac{v \cdot u}{c^2}}$$

Introduce Four-vector:

$$x^0 \equiv ct$$

$$x^1 = x$$

$$x^2 = y$$

$$x^3 = z$$

$$\tilde{x}^0 = \gamma(x^0 - \beta x^1)$$

$$\tilde{x}^1 = \gamma(x^1 - \beta x^0)$$

$$\tilde{x}^2 = x^2$$

$$\tilde{x}^3 = x^3$$

OR, in matrix form:

$$\begin{pmatrix} \tilde{x}^0 \\ \tilde{x}^1 \\ \tilde{x}^2 \\ \tilde{x}^3 \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix}$$

where

$$\tilde{X} = \Lambda \cdot X$$

X is 4-vector Λ is transform

X is 4-vector, Λ is transform matrix. Scalar product:

$$\overline{A \cdot B} = A_x \cdot B_x + A_y \cdot B_y + A_z \cdot B_z = \text{const}$$

(frame invariant)

Two events, separated by time Δt and distance d (in some frame)

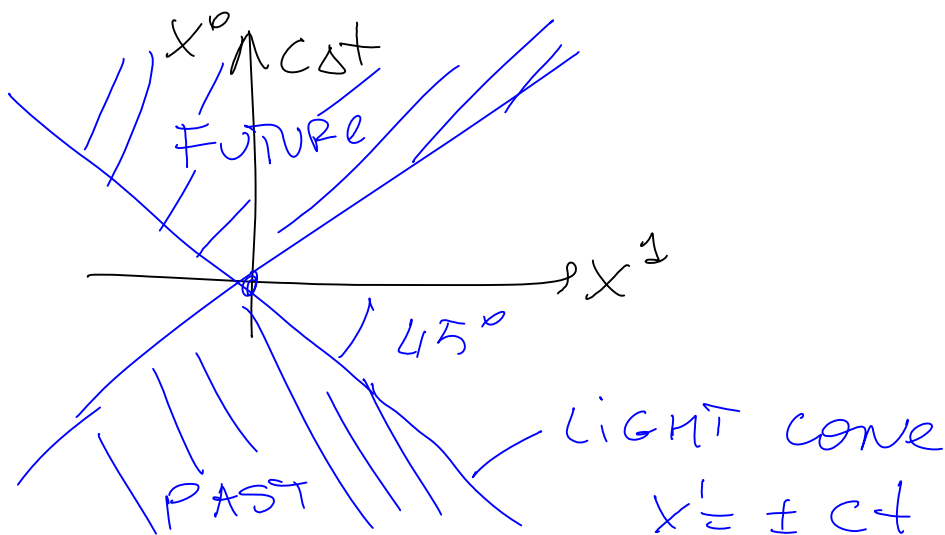
$$I \equiv -c^2 \Delta t^2 + d^2 = \text{const}$$

(frame-invariant)

$I < 0$ timelike
(Δt large, d small)

$I > 0$ spacelike

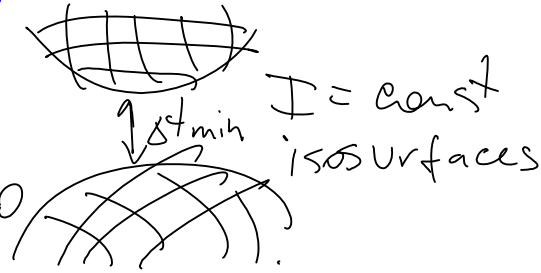
$I = 0$ lightlike



3-D "hypercone"

$$I < 0$$

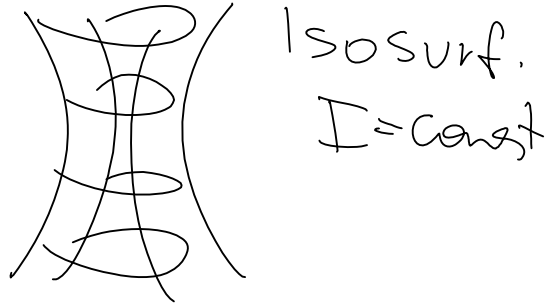
Δt can't be = 0



$$\Delta t_{\min} = \frac{|I|}{c}$$

$$I > 0$$

sign of Δt
can be reversed

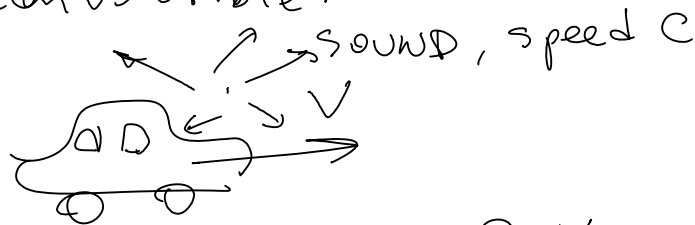


(events A & B can be made simultaneous, or $t_A > t_B$, or $t_B > t_A$, depending on frame).

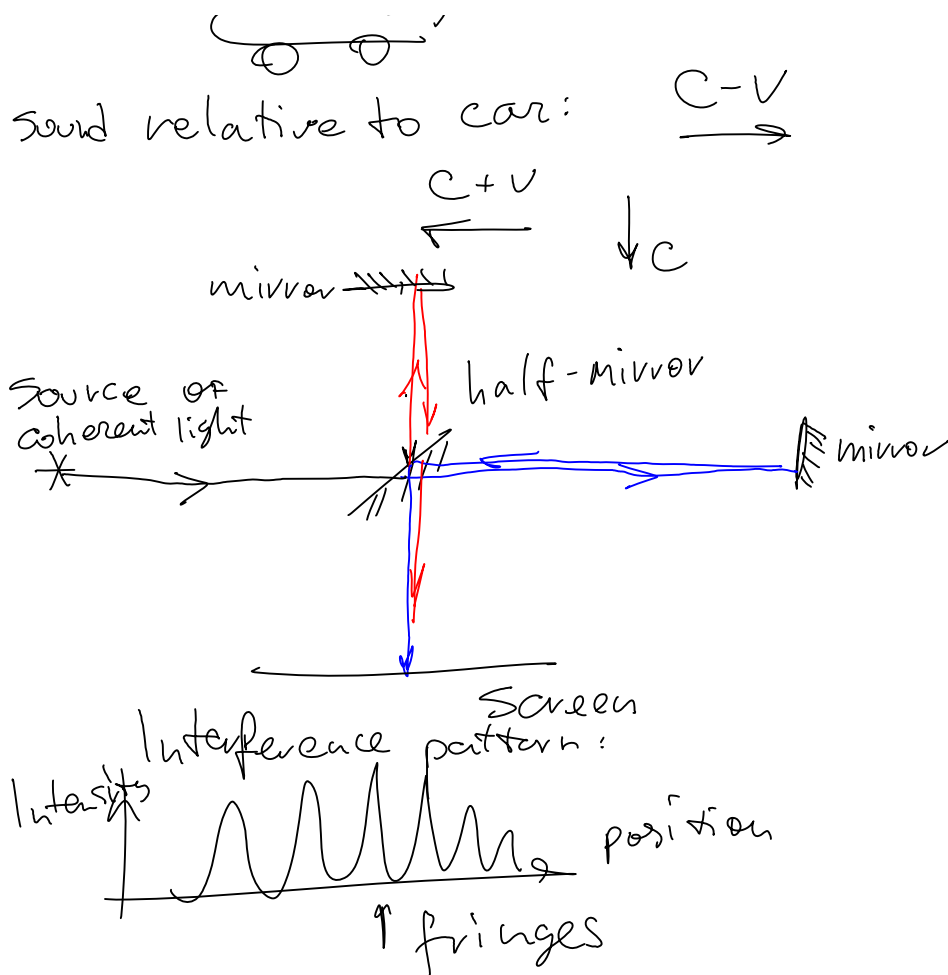
Michelson-Morley experiment:

Idea: measure Earth's velocity with respect to ether.

Analogue: honk while driving a convertible:



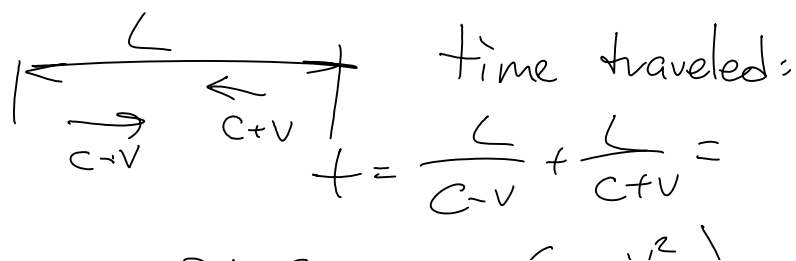
C-V



IF PATH LENGTH DIFFERENCE (PLD) BETWEEN RED & BLUE IS $m \cdot \lambda$ ($m=0,1,2 \dots$ integer)
 \Rightarrow bright spot (constructive)

IF $PLD = m \cdot \lambda + \frac{\lambda}{2}$
 \Rightarrow dark spot (destructive interference)

IF theory of ether is correct



$$= \frac{2L \cdot c}{c^2 - v^2} \approx \frac{2L}{c} \left(1 + \frac{v^2}{c^2}\right)$$

If one arm is \parallel to earth motion, another \perp , rotating them by 90° will change PLD by $4L \cdot \frac{v^2}{c^2}$ (one gets longer by $2L \cdot \frac{v^2}{c^2}$, another shorter by the same amount).

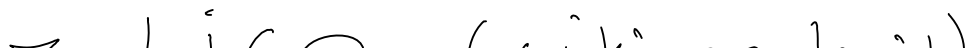
Earth orbital speed is $\approx c \cdot 10^{-4}$
 For $L = 1 \text{ m}$, $\text{PLD} \approx 4 \cdot 10^{-8} \text{ m}$
 OR 40 nm , $\frac{1}{15}$ th of λ_{red} .

By doing multipass approach total length of pass $2L$ was $\sim 11 \text{ m}$ in Michelson-Morley exp. (~ 0.4 fringe width shift was expected).

Observed: zero, with precision $\frac{1}{20} - \frac{1}{40}$ of expected shift.

Ether theory is wrong!

Large-scale, modern version of Michelson-Morley experiment

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Is LIGO (wiki-google it),
search for gravity waves.