PHYS 100C, LECTURE #15

Tuesday, May 04, 2010
12:07 PM

* Jefimenko Equations

"the other" Oleg

given ρ, J for all times, everywhere
find E and B:

$$V(r,t) = \frac{1}{4\pi\varepsilon_0} \cdot \int \frac{\rho(r', t_R)}{\mathscr{r}}\, d\tau'$$

$$A(r,t) = \frac{\mu_0}{4\pi} \int \frac{J(r', t_R)}{\mathscr{r}}\, d\tau'$$

$$E = -\nabla V - \frac{\partial A}{\partial t} \quad \text{and}$$

$$B = \nabla \times A$$

$$\nabla V = \frac{1}{4\pi\varepsilon_0} \cdot \int \left(-\frac{\dot{\rho}}{c} \cdot \frac{\hat{\mathscr{r}}}{\mathscr{r}} - \rho \frac{\hat{\mathscr{r}}}{\mathscr{r}^2} \right) d\tau'$$

(see Lecture #7 Notes)

$$\frac{\partial A}{\partial t} = \frac{\mu_0}{4\pi} \cdot \int \frac{\dot{J}}{\mathscr{r}}\, d\tau'$$

$$E = \frac{1}{4\pi\varepsilon_0} \int \left(\frac{\rho}{\mathscr{r}^2} \hat{\mathscr{r}} + \frac{\dot{\rho}}{c\mathscr{r}} \hat{\mathscr{r}} - \frac{\dot{J}}{c^2 \mathscr{r}} \right) d\tau'$$

Where ρ, $\dot{\rho}$ and $\dot{J}$ evaluated at t_R

Note that only the first term is Coulomb Eq. in retarded formalism, the terms with $\dot{\rho}$ and $\dot{J}$ are

"Unexpected"! ("from" naive approach)

Similarly, $B = \nabla \times A$

$$\nabla \times A = \frac{\mu_0}{4\pi} \int \left[\frac{1}{\imath} (\nabla \times J) - J \times \nabla \left(\frac{1}{\imath} \right) \right] d\tau'$$

(See $\nabla \times (f\vec{A})$ in "Vector identities")

$\nabla \times J$ has terms like

$$\frac{\partial J_z}{\partial y} = \frac{\partial J_z}{\partial t_R} \cdot \frac{\partial t_R}{\partial y}$$

but $t_R = t - \imath/c \Rightarrow \partial t_R = -\frac{1}{c} \partial \imath$

$$\frac{\partial J_z}{\partial y} = \dot{J}_z \cdot \left(-\frac{1}{c} \right) \cdot \frac{\partial \imath}{\partial y}$$

these are now components of

$\dot{J} \times (\nabla \imath)$ (TRUST BUT VERIFY!)

$\nabla \imath = \hat{\imath}$ and $\nabla \left(\frac{1}{\imath} \right) = -\frac{\hat{\imath}}{\imath^2}$

$$B = \frac{\mu_0}{4\pi} \int \left(\frac{J}{\imath^2} + \frac{\dot{J}}{c\imath} \right) \times \hat{\imath} \cdot d\tau'$$

Hand-waving (naive) argument
(also the <u>best</u> kind of argument!)
would give us the first term
for B in "retarded" time form,
but not the $\dot{J}$ term.

Bottom Line: Time retarded ARGUMENT works for potentials V and A, but not for E & B. Expressions for E & B (derived from retarded potentials) include "non-obvious" contributions $\sim \dot{\rho}$ and $\dot{J}$

* Point Charges, Liénard-Wiechert Potentials

As before, $V(r,t) = \frac{1}{4\pi\varepsilon_0} \cdot \int \frac{\rho(r, t_r)}{\mathscr{r}} d\tau'$

If we consider a single "point" charge q moving along some known trajectory $\vec{\omega}(t)$, then a naive person may decide that

$$\rho(r, t_R) = \delta^3(r - \omega(t_R)) \cdot q$$

and therefore

$$V(\mathscr{r}, t) = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{\mathscr{r}}$$

Where $\mathscr{r}$ is retarded distance from $\vec{\omega}(t_R)$ to observer.

This is WRONG!

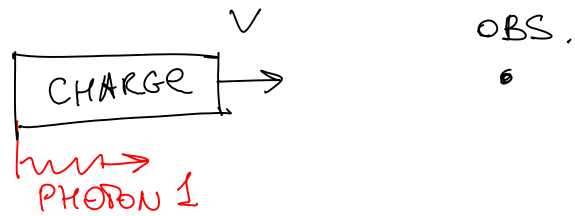
(bad day for naive people!)

The charge is going to be larger than q, by a factor $(1 - \vec{v}\cdot\hat{\mathscr{r}}/c)^{-1}$

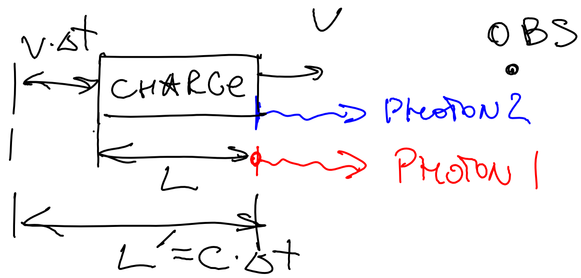
Why? Geometry/kinematics:

$t=0$

CARTOON:



$t=\Delta t$



Observer will "see" photons 1 & 2 at the same time and decide that actual "length" of charge is

$$L' = L + v \cdot \Delta t = c \cdot \Delta t$$

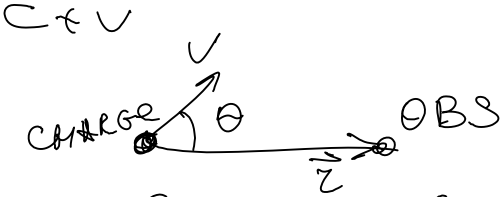
and since $\Delta t = \dfrac{L}{c-v}$

$$L' = L + \frac{Lv}{c-v} = \frac{Lc}{c-v}$$

If charge is moving away from observer,

$$L' = \frac{Lc}{c+v} \quad (\text{"shrinkage"})$$

Generally



$$L' = L \cdot \frac{c}{c - v \cdot \cos\theta} = L \cdot \frac{1}{1 - \vec{v} \cdot \hat{z}/c}$$

The "volume" of charge and total

The "volume" of charge and total charge will therefore be multiplied by $\frac{1}{1-\vec{v}\cdot\hat{r}/c}$

* Extreme CASE: $v=c$ and moving towards observer.
Observer won't see fields/photons, until ALL OF A SUDDEN they all come in at once! ~~together~~ with charge)

General Expression for V:

$$V(r,t) = \frac{1}{4\pi\varepsilon_0} \cdot \frac{qc}{rc - \vec{r}\cdot\vec{v}}$$

AND FOR A, similarly, since $\vec{J} = \rho\vec{v}$:

$$A(r,t) = \frac{\mu_0}{4\pi} \cdot \frac{qc\vec{v}}{rc - \vec{r}\cdot\vec{v}} = \frac{\vec{v}}{c^2} V(r,t)$$

These are Liénard-Wiechert potentials.