PHYS 100C, Lecture 24, May 26th

Wednesday, May 26, 2010
4:09 PM

2nd Newton's Law:

$$F = \frac{\partial p}{\partial t} \quad \text{or}$$

using work: $W \equiv \int F \cdot dl = \int F \cdot u \cdot dt$

$$W = \int \frac{\partial p}{\partial t} \cdot u \cdot dt$$

$$\frac{\partial p}{\partial t} = \frac{\partial}{\partial t}\left(\frac{m\bar{u}}{(1-u^2/c^2)^{1/2}}\right) = \frac{m\frac{\partial \bar{u}}{\partial t}}{(1-u^2/c^2)^{1/2}} -$$

$$-\frac{m}{2}\cdot\frac{\left(-2u\cdot\frac{\partial u}{\partial t}\right)\cdot\bar{u}}{(1-u^2/c^2)^{3/2}} = \frac{m\left(\frac{\partial \bar{u}}{\partial t} - \cancel{\frac{u^2}{c^2}\cdot\frac{\partial \bar{u}}{\partial t}} + \cancel{\frac{u^2}{c^2}\cdot\frac{\partial \bar{u}}{\partial t}}\right)}{(1-u^2/c^2)^{3/2}} =$$

$$= \frac{m\,\partial u/\partial t}{(1-u^2/c^2)^{3/2}}$$

Note that when defined:

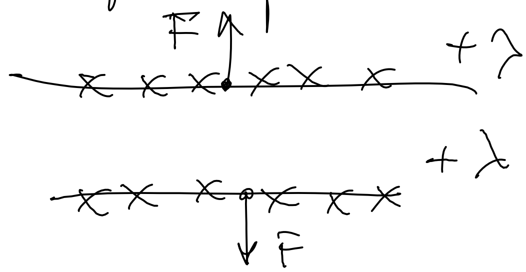
$$E = \frac{mc^2}{(1-u^2/c^2)^{1/2}} \Rightarrow \frac{\partial E}{\partial t} = \frac{m u\cdot\frac{\partial u}{\partial t}}{(1-u^2/c^2)^{3/2}} =$$

$$= \frac{\partial p}{\partial t}\cdot u$$

Big revelation:

Magnetism is relativistic form of electricity.

Thought experiment:



two lines of charge will repell each other. No B-field.

Now get into a frame moving with velocity v.

Now charge density is

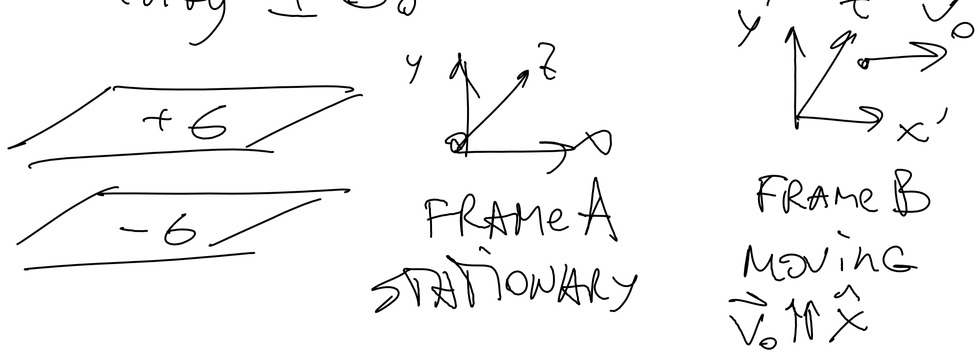
$$\lambda^* = \gamma \cdot \lambda > \lambda$$

increase due to Lorentz contraction.

Coulomb repulsion is stronger as we go faster!

To account for changes, we have to include MAGN. field (wasn't there in stationary frame of reference).

Consider capacitor charge density $\pm \sigma_0$



In A: $E_y^A = \frac{\sigma_0}{\epsilon_0}$ $\quad E_x = E_z = 0$

$V_0 \parallel x$

In A: $E_y^A = \frac{\sigma_0}{\varepsilon_0}$ $E_x = E_z = 0$

In B: $E_y^B = \frac{\sigma}{\varepsilon_0}$ where $\sigma = \gamma \cdot \sigma_0$

since charge density increases due to Lorentz contraction.

$E_x^B = E_z^B = 0$, still.

In B we have also MAGN. field due to currents:

$$\vec{K}_\pm = \pm \sigma V_0 \cdot \hat{x} = \pm \gamma_0 \sigma_0 V_0 \cdot \hat{x}$$

$$B_z^B = -\mu_0 \gamma_0 \sigma V_0$$

$(B^A = 0)$ $\gamma_0 = \frac{1}{(1 - V_0^2/c^2)^{1/2}}$

How do arbitrary fields $\vec{E}, \vec{B}$ transform from one FRAME to another?

In FRAME B we have $\vec{E}, \vec{B}$.

Introduce frame C, moving with v w.r.to B and $u = \frac{v + V_0}{1 + \frac{vV_0}{c^2}}$ w.r.to A

Consider frame C relative to A, fields transform:

$$E_y^C = \frac{\sigma^C}{\epsilon_0} = \frac{\gamma\sigma_0}{\epsilon_0}$$

where $\gamma = \dfrac{1}{(1-u^2/c^2)^{1/2}}$

Also

$$B_z^C = -\mu_0\gamma\sigma_0\cdot u$$

and: $\dfrac{E_y^C}{E_y^B} = \dfrac{\gamma}{\gamma_0}$

$$\frac{\gamma}{\gamma_0} = \left(\frac{1-v_0^2/c^2}{1-u^2/c^2}\right)^{1/2} = \left[\frac{c^2-v_0^2}{c^2-\left(\frac{v_0+v}{1+\frac{v_0 v}{c^2}}\right)^2}\right]^{1/2} =$$

{a few steps skipped}

$$= \frac{1+vv_0/c^2}{\sqrt{1-v^2/c^2}} = \gamma\left(1+\frac{vv_0}{c^2}\right)$$

Note that $v_0 = -\dfrac{B_z^B}{\gamma\mu_0\sigma_0}$

$$E_y^C = \gamma\left(1-\frac{B_z^B\cdot v}{\gamma\mu_0\sigma_0}\cdot\frac{1}{c^2}\right)\cdot\underbrace{\frac{\gamma\sigma_0}{\epsilon_0}}_{E_y^B} =$$

$$= \gamma\left(E_y^B - \frac{v B_z^B}{\underbrace{\mu_0 \varepsilon_0 c^2}_{=1}}\right) = \gamma\left(E_y^B - v B_z^B\right)$$

Similarly:

$$B_z^C = -\frac{\gamma}{\gamma_0} \cdot \mu_0 \gamma_0 \sigma u = \gamma\left(1 + \frac{v v_0}{c^2}\right) \cdot \mu_0 \gamma_0 \sigma u =$$

$$= \gamma B_z^B - \gamma \underbrace{\mu_0 \varepsilon_0}_{1/c^2} v E_y^B = \gamma\left(B_z^B - \frac{v}{c^2} E_y^B\right)$$

Similarly for E_z / B_y
(rotate xy into xz capacitor):

$$E_z^C = \gamma\left(E_z^B + v B_y^B\right)$$

$$B_y^C = \gamma\left(B_z^B + \frac{v}{c^2} E_z^B\right)$$

Parallel to motion (x-axis):

$$E_x^B = E_x^C = E_x^A$$

(capacitor planes in yz produces the same σ^I and has no change of E_x).

Magnetic field stays the same too: $B_x^B = B_x^C = B_x$

* Bottom line message:
E, B fields ⊥ to motion

get transformed into each other,
but not $E, B \parallel$ to motion.
pure E in one F.O.R. can become
E and B in another F.O.R. etc.