PHYS 100C , LECTURE 16

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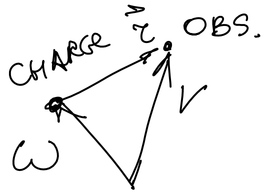
$$V(r,t) = \frac{1}{4\pi\varepsilon_0}\cdot\frac{qc}{\mathscr{r}c - \vec{\mathscr{r}}\cdot\vec{v}} \qquad A(r,t) = \frac{v}{c^2}V(r,t)$$

$$E = -\nabla V - \frac{\partial A}{\partial t} \qquad B = \nabla\times A$$



$$\vec{\mathscr{r}} = \vec{r} - \vec{\omega}$$

$$t - t_R = \frac{|\vec{r} - \vec{\omega}|}{c} \left(= \frac{|\mathscr{r}|}{c} \right)$$

$$\nabla V = \frac{qc}{4\pi\varepsilon_0}\frac{(-1)}{(\mathscr{r}c - \vec{\mathscr{r}}\cdot\vec{v})^2}\nabla(\mathscr{r}c - \vec{\mathscr{r}}\cdot\vec{v})$$

$$\nabla(\mathscr{r}c) = c\cdot\nabla\mathscr{r} = -c\nabla t_R$$

$$\nabla(\vec{\mathscr{r}}\cdot\vec{v}) = \underbrace{(\mathscr{r}\cdot\nabla)v}_{\#1} + \underbrace{(v\cdot\nabla)\mathscr{r}}_{\#2} + \underbrace{\mathscr{r}\times(\nabla\times v)}_{\#3} + \underbrace{v\times(\nabla\times\mathscr{r})}_{\#4}$$

#1: $(\mathscr{r}\cdot\nabla)v = \mathscr{r}_x\cdot\frac{\partial v}{\partial x} + \ldots$

$$\mathscr{r}_x\cdot\frac{\partial v}{\partial x} = \mathscr{r}_x\cdot\underbrace{\frac{\partial v}{\partial t_R}}_{a}\cdot\frac{\partial t_R}{\partial x}$$

$$(\mathscr{r}\cdot\nabla)v = a(\vec{\mathscr{r}}\cdot\nabla t_R)$$

#2 $(v\cdot\nabla)\mathscr{r} = (v\cdot\nabla)(r-\omega) = (v\cdot\nabla)r - (v\cdot\nabla)\omega$

$$(v\cdot\nabla)r = v_x\cdot\frac{\partial\vec{r}}{\partial x} + \ldots = v_x\cdot\hat{x} + v_y\hat{y} + v_z\hat{z} = \vec{v}$$

$$(v\cdot\nabla)\omega = v\,(v\cdot\nabla t_R) \qquad (\text{see } \#1)$$

#3 $\nabla\times v = \frac{\partial v_z}{\partial y}\cdot\hat{x} + \ldots$

$$\frac{\partial v_z}{\partial y}\cdot\hat{x} = \underbrace{\frac{\partial v_z}{\partial t_R}}_{a}\cdot\frac{\partial t_R}{\partial y}\cdot\hat{x}$$

$$(\nabla\times v) = -a\times(\nabla t_R)$$

#4 $\nabla\times\mathscr{r} = \nabla\times(r-\omega) = \nabla\times r - \nabla\times\omega$

$\nabla \times r = 0$

$\nabla \times \omega = -v \times (\nabla t_R)$ (See #3)

Plug #1 - #4 in:

$$\nabla(\tau \cdot v) = a(\bar{\tau} \cdot \nabla t_R) + v - v(v \cdot \nabla t_R) - \tau \times (a \times \nabla t_R) + v \times (v \times \nabla t_R)$$

$$\tau \times (a \times \nabla t_R) = a(\bar{\tau} \cdot \nabla t_R) - \nabla t_R (\tau \cdot a)$$

$$v \times (v \times \nabla t_R) = v(v \cdot \nabla t_R) - \nabla t_R \cdot v^2$$

$$\nabla(\tau \cdot v) = v - \nabla t_R (\tau \cdot a) - \nabla t_R \cdot v^2$$

$$\nabla V = \frac{qc}{4\pi\varepsilon_0} \frac{1}{(\tau c - \bar{\tau} \cdot \bar{v})^2} \cdot \left(v - (c^2 - v^2 + \tau \cdot a)\nabla t_R\right)$$

$\nabla t_R = ?$

$$-c \nabla t_R = \nabla \tau = \nabla\left(\sqrt{\bar{\tau}^2}\right) = \frac{1}{2} \frac{1}{\sqrt{\tau^2}} \cdot \nabla(\bar{\tau} \cdot \bar{\tau}) =$$

$$= \frac{1}{|\tau|} \cdot \left[(\tau \cdot \nabla)\tau + \tau \times (\nabla \times \tau)\right]$$

$$(\tau \cdot \nabla)\tau = (\tau \cdot \nabla) r - (r \cdot \nabla)\omega = \tau - v(\tau \cdot \nabla t_R)$$

$$\nabla \times \tau = v \times \nabla t_R$$

$$-c \nabla t_R = \frac{1}{|\tau|}\left[\bar{\tau} - (\tau \cdot \nabla)\nabla t_R\right]$$

$$\nabla t_R = -\frac{\bar{\tau}}{\tau c - \bar{\tau} \cdot \bar{v}}$$

$$\nabla V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{qc}{(\tau c - \bar{\tau} \cdot \bar{v})^3} \cdot \left((\tau c - \bar{\tau} \cdot \bar{v}) \cdot \bar{v} - (c^2 - v^2 + \bar{\tau} \cdot \bar{a})\tau\right)$$

$$\frac{\partial A}{\partial t} = \frac{1}{4\pi\varepsilon_0} \frac{qc}{(\tau c - \bar{\tau} \cdot \bar{v})^3} \left[(\tau c - \bar{\tau} \cdot \bar{v})\left(-v + \frac{\tau a}{c}\right) + \frac{\tau}{c}(c^2 - v^2 \cdot \tau a)v\right]$$

$$u = c\hat{\tau} - \bar{v}$$

$$E(r,t) = \frac{q}{4\pi\varepsilon_0} \cdot \frac{\imath}{(\imath \cdot u)^3} \left[(c^2 - v^2) u + \bar{\imath} \times (\bar{u} \times \bar{a}) \right]$$

$$\nabla \times A = \frac{1}{c^2} \nabla \times (\bar{V} \cdot v) = \frac{1}{c^2} \left[\bar{V} (\nabla \times v) - v \times (\nabla \times \bar{V}) \right]$$

$$\nabla \times A = -\frac{1}{c} \cdot \frac{q}{4\pi\varepsilon_0} \cdot \frac{1}{(u \cdot \imath)^3} \cdot \imath \times \left[(c^2 - v^2) \bar{v} + (\imath \cdot a) \bar{v} + (\imath \cdot u) a \right]$$

$$B(r,t) = \frac{1}{c} \hat{\imath} \times \bar{E}(\bar{\imath}, t)$$

if $\quad v = 0 \quad a = 0 \quad E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{\imath^2} \cdot \hat{\imath}$

Total force on charge:

$$F = \frac{qQ}{4\pi\varepsilon_0} \cdot \frac{\imath}{(\imath \cdot u)^3} \Big[(c^2 - v^2) u + \imath \times (u \times a) +$$

$$+ \frac{V_Q}{c} \times \left[\hat{\imath} \times \left[(c^2 - v^2) u + \vec{\imath} \times (\vec{u} \times \vec{a}) \right] \right] \Big]$$