

# PHYS 100C, LECTURE 3

Tuesday, April 07, 2009  
8:00 AM

FROM Lecture #2:

\* FOR NORMAL incidence,

INCIDENT (I) wave:

$$\vec{E}_I(z,t) = E_{0I} e^{i(k_1 z - \omega t)} \hat{x}$$

$$\vec{B}_I(z,t) = \frac{E_{0I}}{v_1} e^{i(k_1 z - \omega t)} \hat{y}$$

Where  $v_1 = \frac{1}{\sqrt{\epsilon_1 \mu_1}}$        $k_1 = \frac{\omega}{v_1}$

\* REFLECTED wave: (R)

$$\vec{E}_R(z,t) = E_{0R} e^{i(-k_1 z - \omega t)} \hat{x}$$

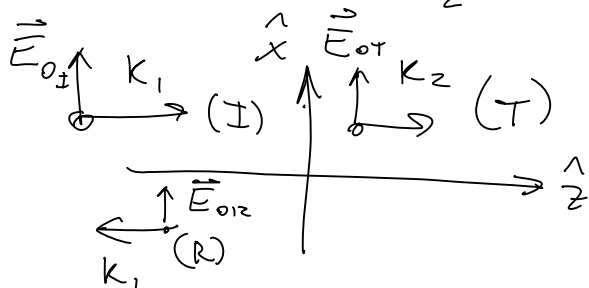
$$\vec{B}_R(z,t) = -\frac{E_{0R}}{v_1} e^{i(-k_1 z - \omega t)} \hat{y}$$

Note "-" sign for  $k_1$  in exp. as well as for  $\vec{B}_R$  direction.

\* TRANSMITTED wave: (T)

$$\vec{E}_T(z,t) = E_{0T} e^{i(k_2 z - \omega t)} \hat{x}$$

$$\vec{B}_T(z,t) = \frac{E_{0T}}{v_2} e^{i(k_2 z - \omega t)} \hat{y}$$



BOUNDARY CONDITIONS  
FOR  $z=0$

$$E_{oI} + E_{oR} = E_{oT} \quad (\text{iii})$$

$$\frac{1}{\mu_1} \left( \frac{1}{v_1} E_{oI} - \frac{1}{v_1} E_{oR} \right) = \frac{1}{\mu_2} \left( \frac{1}{v_2} E_{oT} \right) \quad (\text{iv})$$

(i) AND (ii) are NOT USEFUL  
SINCE  $E^\perp = B^\perp = 0$  (NORMAL INCIDENCE)

(iv) REWRITTEN:

$$E_{oI} - E_{oR} = \frac{\mu_1 v_1}{\mu_2 v_2} E_{oT} \quad (\text{iv})$$

\*NOTE: IF WE MULTIPLY (iii) BY (iv)

$$E_{oI}^2 - E_{oR}^2 = \frac{\mu_1 v_1}{\mu_2 v_2} E_{oT}^2, \text{ OR:}$$

$$\frac{1}{\mu_1 v_1} E_{oI}^2 = \frac{1}{\mu_1 v_1} E_{oR}^2 + \frac{1}{\mu_2 v_2} E_{oT}^2$$

$$\frac{1}{2} \epsilon_1 v_1 E_{oI}^2 = \frac{1}{2} \epsilon_1 v_1 E_{oR}^2 + \frac{1}{2} \epsilon_2 v_2 E_{oT}^2$$

$\uparrow$  Incident Intensity       $\uparrow$  REFLECTED       $\uparrow$  TRANSM.

ENERGY IS CONSERVED

\* IF WE ADD (i) + (ii);

and use  $\beta = \frac{\mu_1 v_1}{\mu_2 v_2} = \frac{\mu_1 n_2}{\mu_2 n_1}$

$$E_{oT} = \frac{2}{1+\beta} E_{oI}$$

$$E_{oR} = \frac{1-\beta}{1+\beta} E_{oI}$$

If:  $\beta < 1$   $E_{OR} > 0$ ,

REFL. is IN PHASE w/ Incident

$\beta > 1$   $E_{OR} < 0$

REFL. WAVE is  $180^\circ$  out of phase

Since  $\mu \sim \mu_0$   $\beta \approx \frac{v_1}{v_2} = \frac{n_2}{n_1}$

$$E_{OT} \approx \left| \frac{2n_1}{n_1 + n_2} \right| E_{OI}$$

$$E_{OR} \approx \left| \frac{n_1 - n_2}{n_1 + n_2} \right| E_{OI}$$

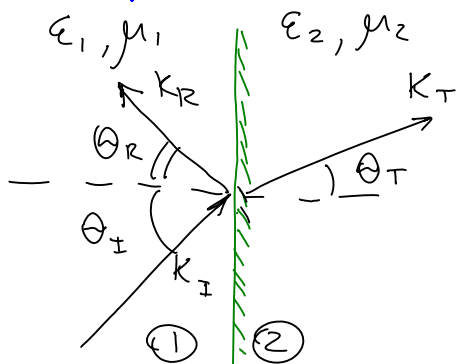
Reflection coefficient:

$$R \equiv \frac{I_R}{I_I} = \left( \frac{E_{OR}}{E_{OI}} \right)^2 = \left( \frac{n_1 - n_2}{n_1 + n_2} \right)^2$$

$$T \equiv \frac{I_T}{I_I} = \left( \frac{E_{OT}}{E_{OI}} \right)^2 \cdot \frac{\epsilon_2 v_2}{\epsilon_1 v_1} \approx \frac{4n_1 n_2}{(n_1 + n_2)^2}$$

\* Note THAT  $R + T = 1$

\* Oblique Incidence CASE:



Incident wave:

$$\vec{E}_I(\vec{r}, t) = \vec{E}_{0I} e^{i(\vec{k}_I \cdot \vec{r} - \omega_I t)}; \vec{B}_I = \frac{(\vec{k}_I \times \vec{E}_I)}{v_1}$$

Reflected:

$$\vec{E}_R(\vec{r}, t) = \vec{E}_{0R} e^{i(\vec{k}_R \cdot \vec{r} - \omega_R t)}; \vec{B}_R = \frac{(\vec{k}_R \times \vec{E}_R)}{v_1}$$

Transmitted:

$$\vec{E}_T(\vec{r}, t) = \vec{E}_{0T} e^{i(\vec{k}_T \cdot \vec{r} - \omega_T t)}; \vec{B}_T = \frac{(\vec{k}_T \times \vec{E}_T)}{v_2}$$

where  $k_I = \frac{\omega_I}{v_1}$ ; etc.

Boundary conditions: ( $z=0$ )

$$(\ ) \cdot e^{i(\vec{k}_I \cdot \vec{r} - \omega_I t)} + (\ ) \cdot e^{i(\vec{k}_R \cdot \vec{r} - \omega_R t)} = (\ ) \cdot e^{i(\vec{k}_T \cdot \vec{r} - \omega_T t)}$$

As Prob. 9.15 shows, this means:

$\omega_I = \omega_R = \omega_T$  (frequencies are the same)  
(we will use  $\omega$ )

and

$$\vec{k}_I \cdot \vec{r} = \vec{k}_R \cdot \vec{r} = \vec{k}_T \cdot \vec{r}$$

$$(k_I)_x \cdot x + (k_I)_y \cdot y = (k_R)_x \cdot x + (k_R)_y \cdot y = (k_T)_x \cdot x + (k_T)_y \cdot y$$

( $z=0$  so it doesn't enter)

FOR  $x=0$ :

$$(k_I)_y = (k_R)_y = (k_T)_y$$

FOR  $y=0$ :

$$(k_I)_x = (k_R)_x = (k_T)_x$$

Let's define  $\hat{x}$  axis so that

$k_I$  is in  $xz$  plane (of incidence)

OR  $(k_I)_y = 0 (= (k_R)_y = (k_T)_y)$

**1<sup>st</sup> Law:**  $\vec{k}_I, \vec{k}_R, \vec{k}_T$  form a "plane of incidence", which also includes SURFACE normal

For x-components:

$$k_I \cdot \sin\theta_I = k_R \cdot \sin\theta_R = k_T \cdot \sin\theta_T$$

Since  $k_I = k_R = \frac{\omega}{v_1}$      $\theta_I = \theta_R$

**2<sup>nd</sup> Law:** Angle of incidence is equal to the angle of reflection

$$\theta_I = \theta_R$$

(Law of Reflection)

For third component,  $k_T = \frac{\omega}{v_2}$

and  $k_I = \frac{\omega}{v_1}$  :

$$\frac{\omega}{v_1} \cdot \sin\theta_I = \frac{\omega}{v_2} \cdot \sin\theta_T$$

$$\frac{\sin\theta_T}{\sin\theta_I} = \frac{v_2}{v_1} = \frac{n_1}{n_2}$$

**3<sup>rd</sup> Law:** Snell's Law  
or Law of Refraction:

$$\frac{\sin\theta_T}{\sin\theta_I} = \frac{n_1}{n_2}$$

\* **BOUNDARY CONDITIONS:**

(exponents cancel out)

$$(i) \quad \epsilon_1 (\tilde{E}_{oI} + \tilde{E}_{oR})_z = \epsilon_2 (\tilde{E}_{oT})_z$$

$$(ii) \quad (\tilde{B}_{oI} + \tilde{B}_{oR})_z = (\tilde{B}_{oT})_z$$

$$(iii) \quad (\tilde{E}_{oI} + \tilde{E}_{oR})_{x,y} = (\tilde{E}_{oT})_{x,y}$$

$$(iv) \quad \frac{1}{\mu_1} (\tilde{B}_{oI} + \tilde{B}_{oR})_{x,y} = \frac{1}{\mu_2} (\tilde{B}_{oT})_{x,y}$$

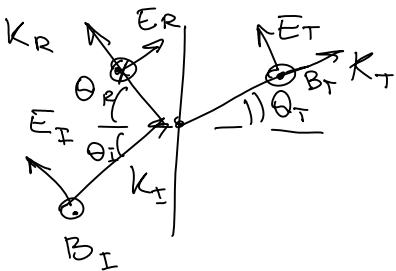
$$\text{and } \tilde{B} = \frac{1}{v} (\bar{k} \times \tilde{E})$$

For p-polarized wave  
( $\tilde{E} \parallel$  plane of incidence):

$$(i) \Rightarrow \epsilon_1 (-\tilde{E}_{oI} \cdot \sin\theta_I + \tilde{E}_{oR} \cdot \sin\theta_R) = -\epsilon_2 \tilde{E}_{oT} \sin\theta_T$$

$$B \perp z, \text{ so (ii): } 0 = 0$$

$$(iii) \quad \tilde{E}_{oI} \cos\theta_I + \tilde{E}_{oR} \cos\theta_R = \tilde{E}_{oT} \cos\theta_T$$



$$(iv) \Rightarrow \frac{1}{\mu_1 v_1} (\tilde{E}_{oI} - \tilde{E}_{oR}) = \frac{1}{\mu_2 v_2} \tilde{E}_{oT}$$

since  $\theta_I = \theta_R$ , (i) & (iv):

$$\tilde{E}_{oI} - \tilde{E}_{oR} = \tilde{E}_{oT} \beta$$

$$\text{where } \beta \equiv \frac{\mu_1 v_1}{\mu_2 v_2} = \frac{\epsilon_2}{\epsilon_1} \frac{\sin\theta_T}{\sin\theta_I} = \frac{\mu_1 n_2}{\mu_2 n_1}$$

.....  $\approx \approx \approx$

$$(iii) \Rightarrow \tilde{E}_{or} + \tilde{E}_{oe} = \tilde{E}_{oi} \cdot \alpha$$

$$\alpha = \frac{\cos \theta_T}{\cos \theta_I}$$

add / subtract: **Fresnel Eqs:**

$$\tilde{E}_{or} = \left( \frac{\alpha - \beta}{\alpha + \beta} \right) \tilde{E}_{oi} ; \tilde{E}_{ot} = \left( \frac{2}{\alpha + \beta} \right) \tilde{E}_{oi}$$

$$\text{since } \cos \theta_T = \sqrt{1 - \sin^2 \theta_T} = \sqrt{1 - \frac{n_1^2}{n_2^2} \sin^2 \theta_I}$$

$$\alpha(\theta_I) = \sqrt{1 - \frac{n_1^2 \sin^2 \theta_I}{n_2^2}} \cdot \frac{1}{\cos \theta_I}$$

$$R \equiv \frac{I_R}{I_I} = \left( \frac{E_{or}}{E_{oi}} \right)^2 = \left( \frac{\alpha - \beta}{\alpha + \beta} \right)^2$$

$$T \equiv \frac{I_T}{I_I} = \frac{\epsilon_2 v_2}{\epsilon_1 v_1} \cdot \left( \frac{E_{ot}}{E_{oi}} \right)^2 \cdot \frac{\cos \theta_T}{\cos \theta_I} = \alpha \beta \left( \frac{2}{\alpha + \beta} \right)^2$$

\* Brewster Angle:

$$\text{when } \alpha = \beta \quad R = 0$$

$$\alpha = \beta \Rightarrow 1 - \left( \frac{n_1}{n_2} \right)^2 \sin^2 \theta_B = \beta^2 \cdot \cos^2 \theta_B = \beta^2 (1 - \sin^2 \theta_B)$$

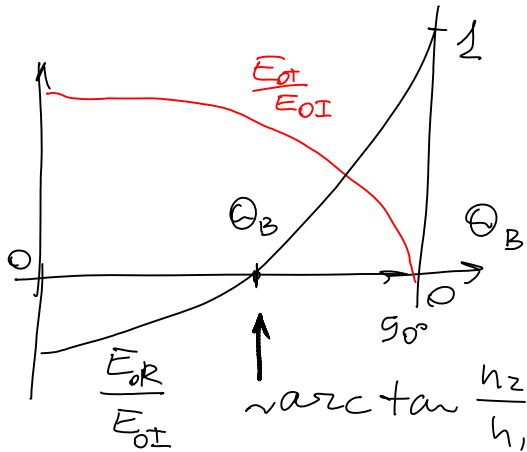
$$\sin^2 \theta_B = \frac{1 - \beta^2}{\left( \frac{n_1}{n_2} \right)^2 - \beta^2} \quad \text{since } \beta \approx \frac{n_2}{n_1}$$

$$\sin^2 \theta_B = \frac{1 - \beta^2}{\left( \frac{1}{\beta} \right)^2 - \beta^2} = \beta^2 \frac{1 - \beta^2}{1 - \beta^4} = \frac{\beta^2}{4\beta^2}$$

$$\text{OR} \quad \tan^2 \theta_B = \frac{\sin^2 \theta_B}{1 - \sin^2 \theta_B} = \beta^2$$

$$1 \sim n.$$

$$\tan \theta_B = \frac{n_2}{n_1}$$



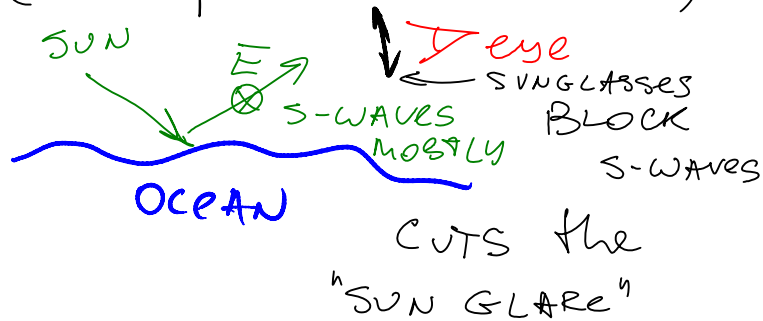
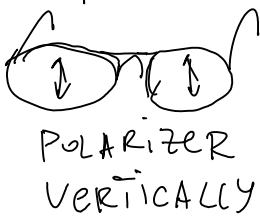
$E_{or}$  is  $< 0$  (out of phase by  $180^\circ$  w.r. to  $E_{oi}$ )

\* Application #1: polarized SUNGLASSES

p-polarized waves  $\Rightarrow$  reflection is suppressed around  $\theta_B$  ( $R=0$  at  $\theta_B$ )

s-polarized waves "survive" reflection

Result: reflected sunlight is preferentially s-polarized ( $E \perp$  plane of incidence)



\* APPLICATION #2

Brewster ANGLE MICROSCOPY:

