Lecture 23, May 24

Monday, May 24, 2010
9:39 PM

"Proper" velocity.

relativistic velocities add in awkward manner:

$$\tilde{u} = \frac{u - V}{1 - \frac{uV}{c^2}} \quad \text{(not linear)}$$

Introduce "proper" velocity:

$$\eta = \frac{\partial l}{\partial \tau} \begin{array}{l} \leftarrow \text{distance in lab frame} \\ \leftarrow \text{time in moving frame} \end{array}$$

$$\eta = \gamma u$$

then $\eta^0 = \gamma \cdot \frac{\partial x^0}{\partial \tau} = c \frac{\partial t}{\partial \tau} = c\gamma$

$$\eta = \begin{pmatrix} c\gamma \\ u\gamma \\ 0 \\ 0 \end{pmatrix}$$

and Lorentz transform. (just like for $x^0, x^1, ..$):

$$\tilde{\eta}^0 = \gamma(\eta^0 - \beta \eta^1)$$

$$\tilde{\eta}^1 = \gamma(\eta^1 - \beta \eta^0)$$

$$\tilde{\eta}^1 = \gamma(\eta^1 - \beta\eta^0)$$
$$\tilde{\eta}^2 = \eta^2$$
$$\tilde{\eta}^3 = \eta^3$$

Introduce "proper" momentum:

$$p = m \cdot \eta = \gamma \cdot m u$$

$$p^0 = m \cdot c\gamma \qquad (\Leftarrow \text{weird})$$

Einstein introduced it as relativistic energy E:

$$E = p^0 \cdot c = \frac{mc^2}{\sqrt{1 - v^2/c^2}} = \gamma \cdot mc^2$$

FOR $v = 0$

$$E = mc^2$$

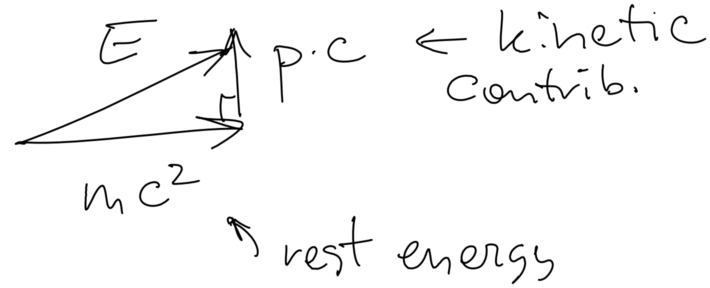
(ANY OF YOU seen this eq. BEFORE? IT'S RATHER OBSCURE!)

$$(p^\mu)^2 = -(p^0)^2 + p^2 = -\gamma^2 m^2 c^2 + \gamma^2 m^2 v^2 =$$
$$= \gamma^2 m^2 c^2 \left(\frac{v^2}{c^2} - 1\right) = m^2 c^2 \leftarrow \text{invariant}$$

$\uparrow$ 4-vector

OR: (since $p^0 = \frac{E}{c}$)

$$E^2 - p^2 c^2 = m^2 c^4$$



For small v,

$$\Delta E = E_{KIN} = \frac{1}{2} m v^2 + \ldots$$

(assuming rest mass does NOT change – no nuclear reactions etc.)

Also, if $m = 0$,

$$E^2 = p^2 c^2 \quad \text{or} \quad E = pc$$

photons: $E = h\nu$

$$p = \frac{h\nu}{c}$$