PHYS 100C, LECTURE #3

Friday, April 02, 2010
8:21 AM

General Expression EM waves

$$\vec{E}(\vec{r},t) = \tilde{E}_0 \, e^{i(\vec{k}\cdot\vec{r}-\omega t)} \, \hat{n}$$

$$\vec{B}(\vec{r},t) = \frac{1}{c}\tilde{E}_0 \, e^{i(\vec{k}\cdot\vec{r}-\omega t)} (\hat{k}\times\hat{n}) =$$

$$= \frac{1}{c}\hat{k}\times\vec{E}$$

$$\hat{n}\cdot\vec{k} = 0 \qquad (\vec{E}\perp\vec{k}, \text{transverse})$$

Energy transport

* Energy density $\left(=\frac{E}{V}\right)$

$$u = \frac{1}{2}\left(\varepsilon_0 E^2 + \frac{1}{\mu_0}B^2\right)$$

$$B = \frac{1}{c}E = \sqrt{\mu_0\varepsilon_0}\,E$$

$$u = \frac{1}{2}\Big(\underbrace{\varepsilon_0 E^2}_{E\text{-part}} + \underbrace{\frac{\cancel{\mu_0}\varepsilon_0}{\cancel{\mu_0}}E^2}_{B\text{-part}}\Big) = \varepsilon_0 E^2$$

EQUAL!

$$u = \varepsilon_0 E_0^2 \cos^2(kz-\omega t+\delta)$$

* Energy transferred:

energy flux density

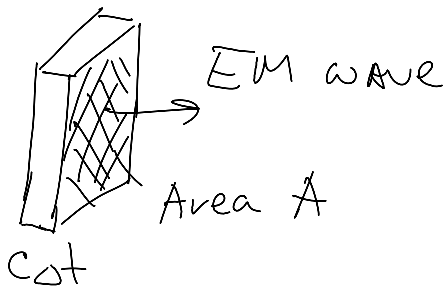
(energy per unit area, per unit time, $\frac{E}{A\cdot\Delta t}$)

Poynting vector:

$$\vec{S} = \frac{1}{\mu_0}(\vec{E} \times \vec{B}) = \frac{1}{\mu_0 c} E_0^2 \cos^2(kz - \omega t + \delta)\hat{z} =$$

$$= c\, u\, \hat{z}$$

Does this make sense?



Energy stored in volume $A \cdot c\Delta t$ is $u \cdot A \cdot c \cdot \Delta t$ will be transferred across A in time Δt

$$|S \text{ (flux density)}| = \frac{\text{energy}}{\text{area} \cdot \text{time}} = \frac{uA \cdot c\Delta t}{A \cdot \Delta t} = uc$$

* Momentum density (momentum/volume)

$$\vec{P} = \frac{\vec{S}}{c^2} = \frac{u}{c}\hat{z}$$

Also makes sense: Photon Energy $\hbar\omega$
Photon Momentum $\frac{\hbar\omega}{c}$

We usually are interested in **average** values of S, P, u, etc.

Red light $\lambda \approx 600\,\text{nm} = 6 \cdot 10^{-7}\,\text{m}$

Period $T = \frac{\lambda}{c} = \frac{6 \cdot 10^{-7}\,\text{m}}{3 \cdot 10^{8}\,\text{m/s}} \sim 2 \cdot 10^{-15}\,\text{s}$

or 2 femtoseconds

Most experiments average over

Most experiments average over many T's.

$$\langle \cos^2 \rangle = \langle \sin^2 \rangle = \frac{1}{2}$$

(since $\cos^2 + \sin^2 = 1$)

$\langle u \rangle = \frac{1}{2}\varepsilon_0 E^2$ energy density

$\langle \bar{S} \rangle = \frac{1}{2} c \varepsilon_0 E_0^2 \hat{z}$ energy flux

$\langle \bar{p} \rangle = \frac{\langle u \rangle}{c} \hat{z}$ momentum density

Experiments usually measure average power per area ≡ intensity

$$I = |\langle \bar{S} \rangle| = \frac{1}{2} c \varepsilon_0 E_0^2$$

Radiation pressure ≡ $\frac{\langle \text{force} \rangle}{\text{area}}$

$$\underset{\text{pressure}}{P} = \frac{1}{A}\left|\frac{\overset{\text{momentum}}{\Delta \bar{p}}}{\delta t}\right| = \frac{1}{A} \cdot \frac{|\langle \bar{p} \rangle| \cdot A c \Delta t}{\Delta t} = |\langle \bar{p} \rangle| \cdot c =$$

$$= \frac{1}{2} \varepsilon_0 E^2 = \frac{I}{c}$$

assuming 100% absorption.

For 100% reflection, double it

$$P = \varepsilon_0 E^2 = \frac{2I}{c}$$

* EM waves in media:

MAXWELL EQS IN MEDIA

NO FREE CHARGE $\rho=0$

NO FREE CURRENTS $j=0$

$$\bar{\nabla}\cdot\bar{D}=0 \qquad (i)$$
$$\bar{\nabla}\cdot\bar{B}=0 \qquad (ii)$$
$$\bar{\nabla}\times\bar{E}=-\partial B/\partial t \qquad (iii)$$
$$\bar{\nabla}\times\bar{H}=\partial D/\partial t \qquad (iv)$$

<u>Linear</u> media $\bar{D}=\varepsilon\bar{E}$

$$\bar{H}=\frac{1}{\mu}\bar{B}$$

<u>Homogeneous</u> μ, ε indep. of x, y, z

$$\bar{\nabla}\cdot\bar{E}=0 \qquad (i)$$
$$\bar{\nabla}\cdot\bar{B}=0 \qquad (ii)$$
$$\bar{\nabla}\times\bar{E}=-\frac{\partial B}{\partial t} \qquad (iii)$$
$$\bar{\nabla}\times\bar{B}=\mu\varepsilon\frac{\partial E}{\partial t} \qquad (iv)$$

SAME AS MAXWELL EQ'S FOR VACUUM

BUT $\mu_0\to\mu$

$\varepsilon_0\to\varepsilon$