Random Close Packing (RCP)

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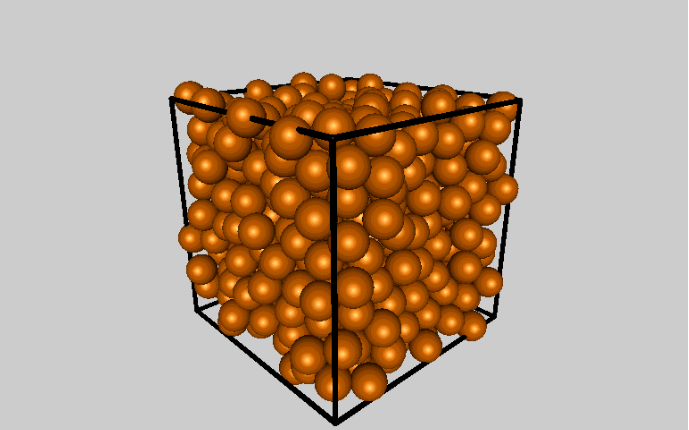
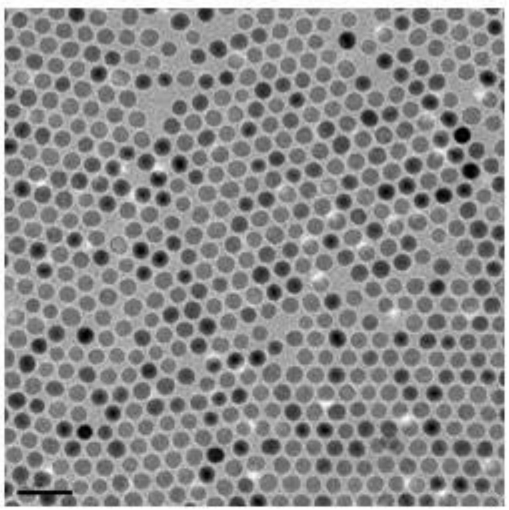
Outline

- Introduction
- Maximum Packing Fraction
  - Ellipsoids, Spheres
- Simulations
- Coordination Number
- Conclusion

Introduction



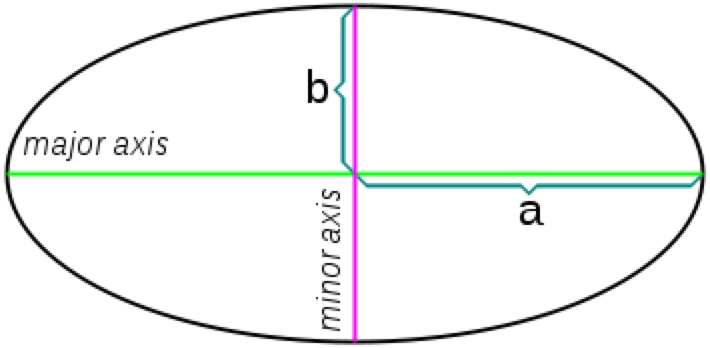


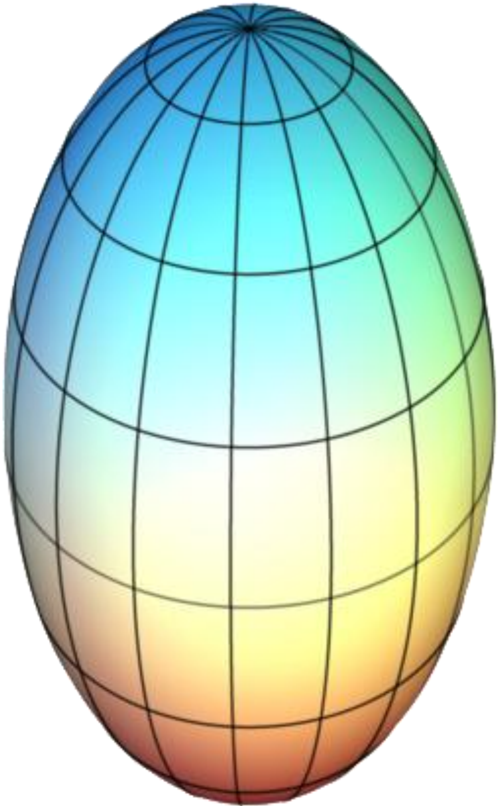
TEM of 15 nm Iron Oxide

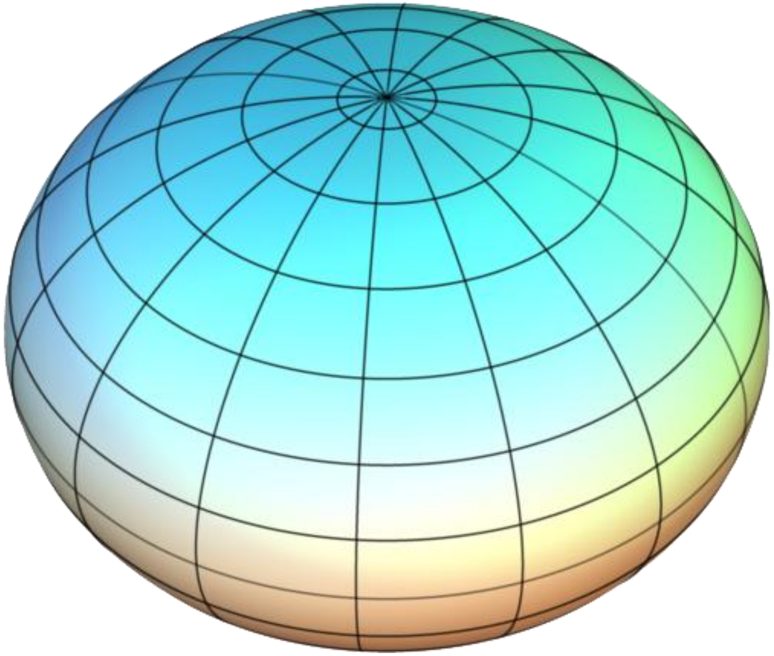
Pictures From Wikimedia Commons, Ocean NanoTech

Maximum Packing Fraction

$$\varphi = \frac{V_{particles}}{V_{occupied}}$$







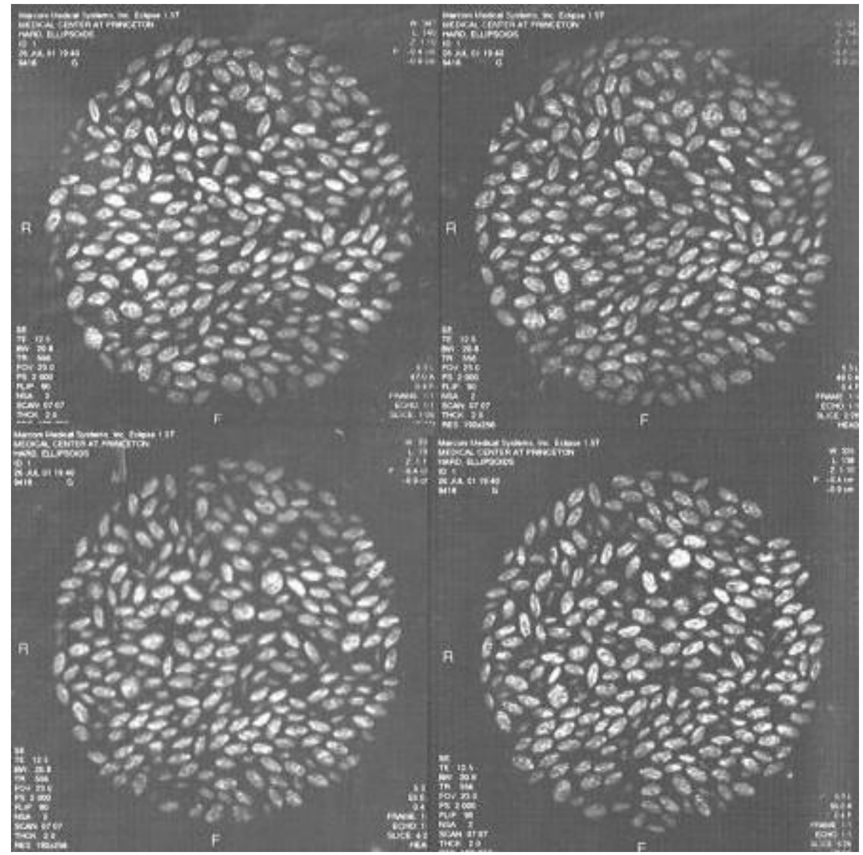
Pictures From Wikimedia Commons

Aside: M&M’s Color Distribution

Color	1997	2011	Change
Red	20%	13%	-7%
Orange	10%	20%	+10%
Yellow	20%	14%	-6%
Green	10%	16%	+6%
Blue	10%	24%	+14%
Brown	30%	13%	-17%

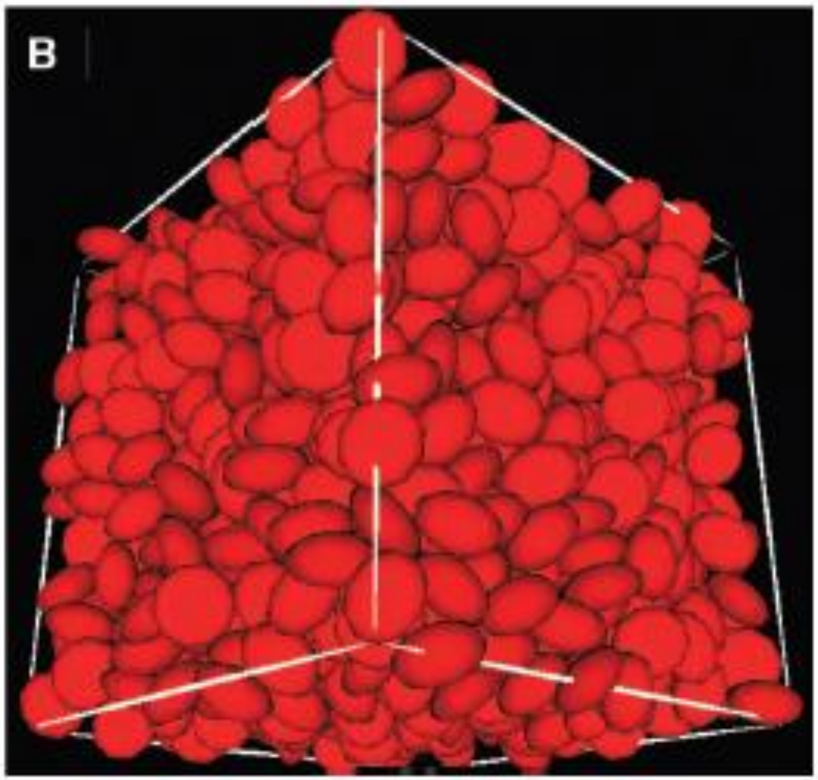
Source: Some internet blog with letters from Mars, Inc.

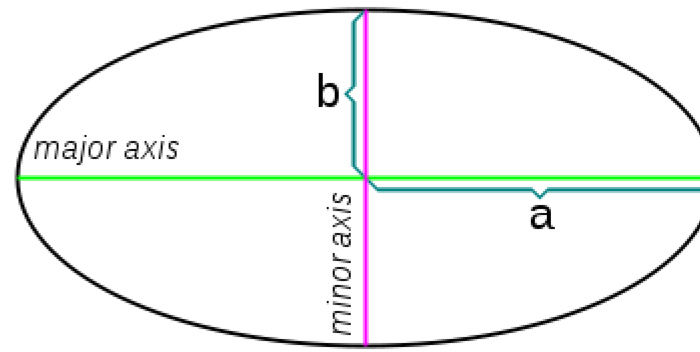
M&M Close Packing



Pictures from Princeton Colloquium by Paul Chaikin

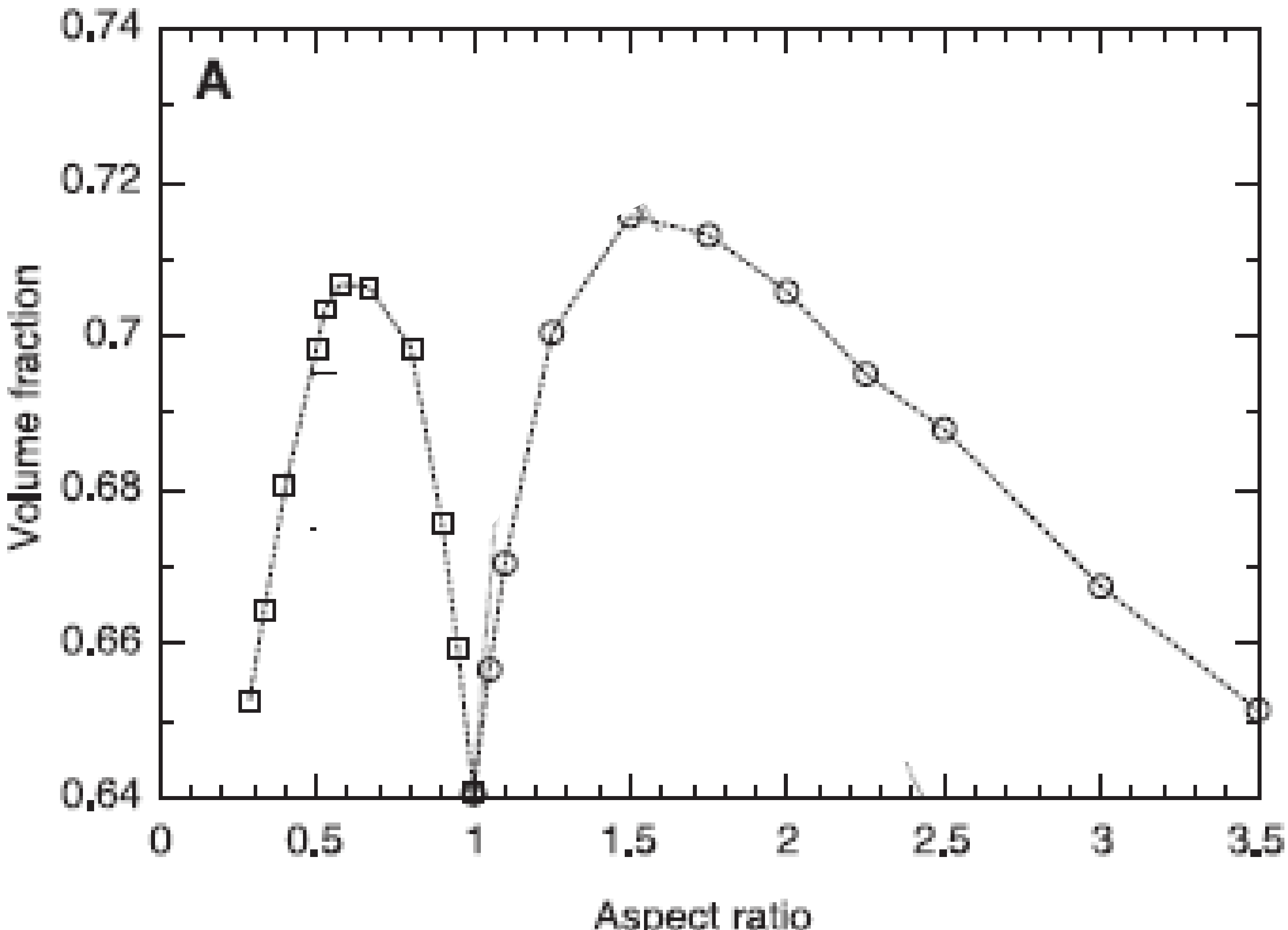
Simulated Close Packing





Donev, *et. al.*, "Improving the Density of Jammed Disordered Packings Using Ellipsoids," Science 303, 990-993 (2004)

Packing Fraction of Spheroids



Donev, *et. al.*, "Improving the Density of Jammed Disordered Packings Using Ellipsoids," Science 303, 990-993 (2004)

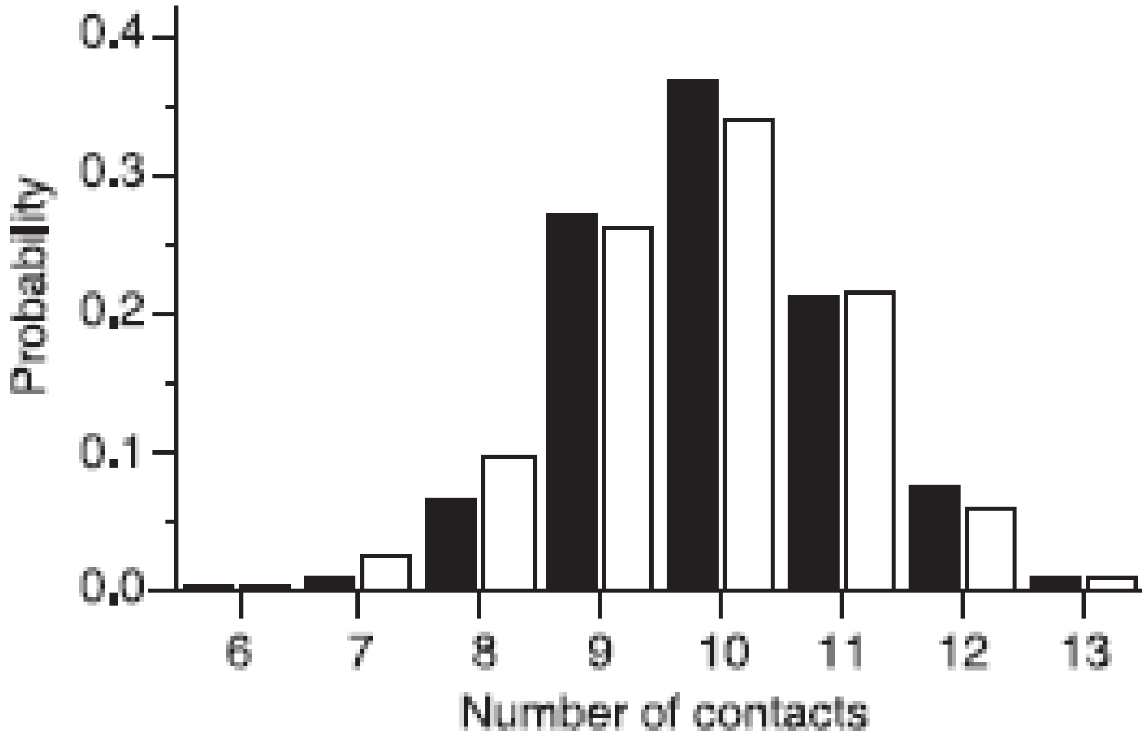
The Isostatic Conjecture

$$Z_{coordination} = 2 * dof$$

- Spheres: Z = 6
- M&M's: Z = 10

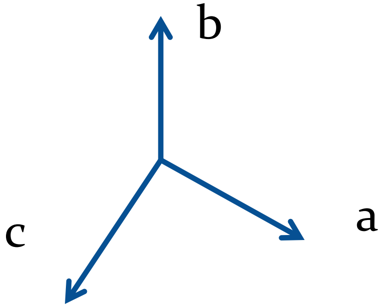
Picture from Princeton Colloquium by Paul Chaikin

Coordination Numbers

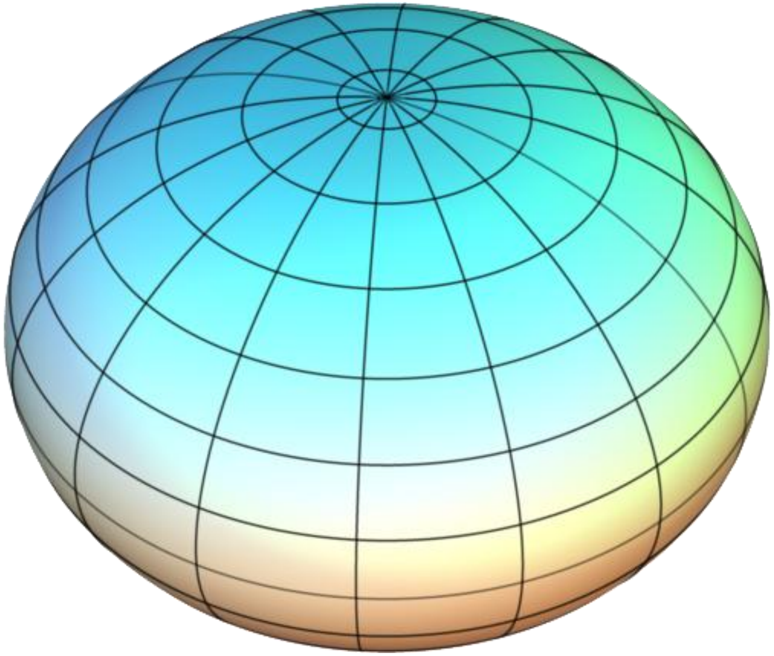


Donev, *et. al.*, "Improving the Density of Jammed Disordered Packings Using Ellipsoids," Science 303, 990-993 (2004)

Aspherical Ellipsoids



$$a \neq b \neq c$$



- 1.25 : 1 : 0.8
  - RCP: 0.743
  - Crystalline: 0.757

Pictures from Wikimedia Commons, Princeton Colloquium by Paul Chaikin

Funding for Advertising?

Improving the Density of Jammed Disordered Packings Using Ellipsoids

Aleksandar Donev,[1,4] Ibrahim Cisse,[2,5] David Sachs,[2]
Evan A. Variano,[2,6] Frank H. Stillinger,[3] Robert Connelly,[7]
Salvatore Torquato,[1,3,4]* P. M. Chaikin[2,4]

Packing problems, such as how densely objects can fill a volume, are among the most ancient and persistent problems in mathematics and science. For equal spheres, it has only recently been proved that the face-centered cubic lattice has the highest possible packing fraction $\varphi = \pi/\sqrt{18} \approx 0.74$. It is also well known that certain random (amorphous) jammed packings have $\varphi \approx 0.64$. Here, we show experimentally and with a new simulation algorithm that ellipsoids can randomly pack more densely—up to $\varphi = 0.68$ to 0.71 for spheroids with an aspect ratio close to that of M&M's Candies—and even approach $\varphi \approx 0.74$ for ellipsoids with other aspect ratios. We suggest that the higher density is directly related to the higher number of degrees of freedom per particle and thus the larger number of particle contacts required to mechanically stabilize the packing. We measured the number of contacts per particle $Z \approx 10$ for our spheroids, as compared to $Z \approx 6$ for spheres. Our results have implications for a broad range of scientific disciplines, including the properties of granular media and ceramics, glass formation, and discrete geometry.

Donev, *et. al.*, "Improving the Density of Jammed Disordered Packings Using Ellipsoids," Science 303, 990-993 (2004), Picture from Princeton Colloquium by Paul Chaikin