PHYS 100C, LECTURE 2

Thursday, April 02, 2009
6:30 AM

General Expression EM waves

$$\vec{E}(\vec{r},t) = \tilde{E}_0 \, e^{i(\vec{k}\cdot\vec{r}-\omega t)} \, \hat{n}$$

$$\vec{B}(\vec{r},t) = \frac{1}{c} \tilde{E}_0 \, e^{i(\vec{k}\cdot\vec{r}-\omega t)} (\hat{k}\times\hat{n}) =$$

$$= \frac{1}{c} \hat{k} \times \vec{E}$$

$\hat{n}\cdot\vec{k} = 0$ ($\vec{E} \perp \vec{k}$, transverse)

Energy transport

* Energy density $\left(= \frac{E}{V}\right)$

$$u = \frac{1}{2}\left(\varepsilon_0 E^2 + \frac{1}{\mu_0} B^2\right)$$

$$B = \frac{1}{c} E = \sqrt{\mu_0 \varepsilon_0}\, E$$

$$u = \frac{1}{2}\Big(\underbrace{\varepsilon_0 E^2}_{E\text{-part}} + \underbrace{\frac{\cancel{\mu_0}\varepsilon_0}{\cancel{\mu_0}} E^2}_{B\text{-part}}\Big) = \varepsilon_0 E^2$$

EQUAL!

$$u = \varepsilon_0 E_0^2 \cos^2(kz - \omega t + \delta)$$

* Energy transferred:

energy flux density

(energy per unit area, per unit time, $\frac{E}{A\cdot\Delta t}$)

Poynting vector:

$$\vec{S} = \frac{1}{\mu_0}\left(\vec{E}\times\vec{B}\right) = \frac{1}{\mu_0 c} E_0^2 \cos^2(kz-\omega t+\delta)\hat{z} =$$

$$= c\, u\, \hat{z}$$

Does this make sense?

Energy stored in volume $A\cdot c\Delta t$ is $u\cdot A\cdot c\cdot\Delta t$ will be transferred across A in time Δt

$$|S\text{(flux density)}| = \frac{\text{energy}}{\text{area}\cdot\text{time}} = \frac{uA\cdot c\Delta t}{A\cdot\Delta t} = uc$$

* Momentum density (momentum/volume)

$$\vec{P} = \frac{\vec{S}}{c^2} = \frac{u}{c}\hat{z}$$

Also makes sense: Photon Energy $\hbar\omega$
Photon Momentum $\frac{\hbar\omega}{c}$

We usually are Interested in **average** values of S, P, u, etc.

Red light $\lambda \approx 600\,\text{nm} = 6\cdot 10^{-7}\,\text{m}$

Period $T = \frac{\lambda}{c} = \frac{6\cdot 10^{-7}\,\text{m}}{3\cdot 10^{8}\,\text{m/s}} \sim 2\cdot 10^{-15}\,\text{s}$

or 2 femtoseconds

Most experiments average over many T's.

$\langle \cos^2 \rangle = \langle \sin^2 \rangle = 1/2$

(since $\cos^2 + \sin^2 = 1$)

$\langle u \rangle = \frac{1}{2}\varepsilon_0 E^2$ energy density

$\langle \bar{S} \rangle = \frac{1}{2} c \varepsilon_0 E_0^2 \hat{z}$ energy flux

$\langle \bar{p} \rangle = \frac{\langle u \rangle}{c} \hat{z}$ momentum density

Experiments usually measure average power per area $\equiv$ intensity

$$I = |\langle \bar{S} \rangle| = \frac{1}{2} c \varepsilon_0 E_0^2$$

Radiation pressure $\equiv \frac{\langle \text{force} \rangle}{\text{area}}$

$$\underset{\text{pressure}}{P} = \frac{1}{A}\left|\frac{\Delta \overset{\text{momentum}}{p}}{\delta t}\right| = \frac{1}{A} \cdot \frac{|\langle \bar{p} \rangle| \cdot A c \Delta t}{\Delta t} = |\langle \bar{p} \rangle| \cdot c =$$

$$= \frac{1}{2} \varepsilon_0 E^2 = \frac{I}{c}$$

assuming 100% absorption.

For 100% reflection, <u>double it</u>

$$P = \varepsilon_0 E^2 = \frac{2I}{c}$$

* EM waves in media:

MAXWELL EQS IN MEDIA

NO FREE CHARGE $\rho = 0$

NO FREE CURRENTS $j = 0$

$$\bar{\nabla}\cdot\bar{D} = 0 \quad (i)$$
$$\bar{\nabla}\cdot\bar{B} = 0 \quad (ii)$$
$$\bar{\nabla}\times\bar{E} = -\partial B/\partial t \quad (iii)$$
$$\bar{\nabla}\times\bar{H} = \partial D/\partial t \quad (iv)$$

Linear media $\bar{D} = \varepsilon\bar{E}$

$\bar{H} = \frac{1}{\mu}\bar{B}$

Homogeneous μ, ε indep. of x, y, t

$$\bar{\nabla}\cdot\bar{E} = 0 \quad (i)$$
$$\bar{\nabla}\cdot\bar{B} = 0 \quad (ii)$$
$$\bar{\nabla}\times\bar{E} = -\frac{\partial B}{\partial t} \quad (iii)$$
$$\bar{\nabla}\times\bar{B} = \mu\varepsilon\frac{\partial E}{\partial t} \quad (iv)$$

SAME AS MAXWELL EQ'S FOR VACUUM

BUT $\mu_0 \to \mu$

$\varepsilon_0 \to \varepsilon$

Solutions: EM WAVES

with velocity $v = \frac{1}{\sqrt{\varepsilon\mu}} = \frac{c}{n}$

Where $n = \sqrt{\frac{\varepsilon\mu}{\varepsilon_0\mu_0}}$ index of refraction

$\mu \simeq \mu_0$ for most materials

$n \simeq \sqrt{\frac{\varepsilon}{\varepsilon_0}} = \sqrt{\varepsilon_R} \leftarrow$ dielectric constant

$\varepsilon_R = \frac{\varepsilon}{\varepsilon_0} > 1$ (almost always)

$$\varepsilon_R = \frac{\varepsilon}{\varepsilon_0} > 1 \quad \text{(almost always)}$$

$$n > 1 \quad \text{and} \quad v < c$$

Similarly, energy density:

$$u = \frac{1}{2}\left(\varepsilon E^2 + \frac{1}{\mu} B^2\right)$$

$$\vec{B} = \frac{1}{v}\,\hat{k} \times \vec{E} \quad \text{(transverse field)}$$

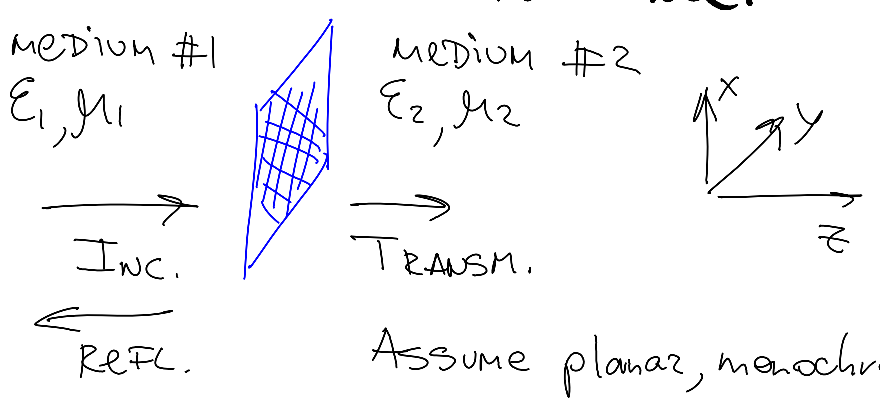
$$\Downarrow$$

$$u = \varepsilon E^2$$

$$\vec{S} = \frac{1}{\mu}\left(\vec{E} \times \vec{B}\right) = v \cdot u \cdot \hat{k}$$

$$I = \left|\langle \vec{S} \rangle\right| = \frac{1}{2}\varepsilon v E_0^2$$

* Reflection/Transmission at NORMAL INCIDENCE:



Assume planar, monochrom. waves

* BOUNDARY CONDITIONS:
FROM MAXWELL Eq. (i)–(iv):

$$D_1^{\perp} = D_2^{\perp} \quad \text{OR} \quad \varepsilon_1 E_1^{\perp} = \varepsilon_2 E_2^{\perp} \quad (i)$$

$$B_1^{\perp} = B_2^{\perp} \quad (ii)$$

$$\bar{E}_1^{\parallel} = \bar{E}_2^{\parallel} \qquad \text{(iii)}$$

$$\bar{H}_1^{\parallel} = \bar{H}_2^{\parallel} \quad \text{OR} \quad \frac{B_1^{\parallel}}{\mu_1} = \frac{B_2^{\parallel}}{\mu_2} \qquad \text{(iv)}$$

* FOR NORMAL INCIDENCE,

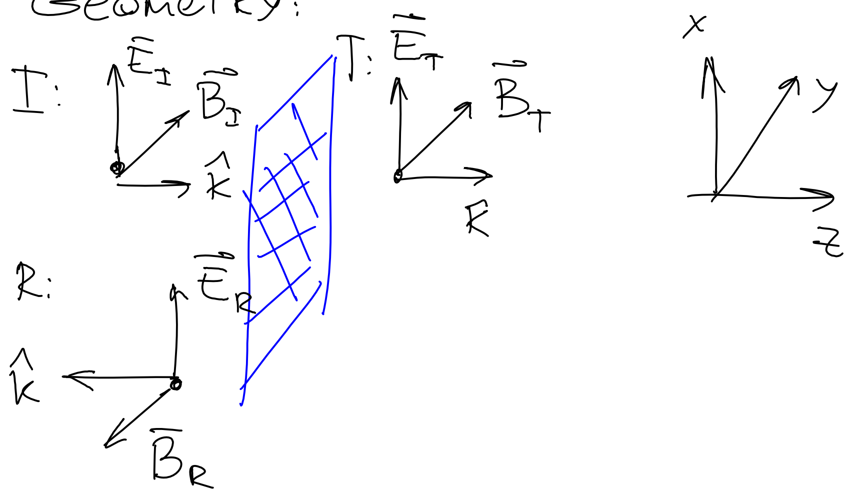
<u>INCIDENT</u> (I) WAVE:

$$\tilde{\vec{E}}_I(z,t) = E_{0I}\, e^{i(k_1 z - \omega t)}\, \hat{x}$$

$$\tilde{\vec{B}}_I(z,t) = \frac{E_{0I}}{v_1}\, e^{i(k_1 z - \omega t)}\, \hat{y}$$

where $v_1 = \frac{1}{\sqrt{\mu_1 \epsilon_1}}$

* GEOMETRY:



* <u>REFLECTED WAVE</u>: (R)

$$\tilde{\vec{E}}_R(z,t) = E_{0R}\, e^{i(-k_1 z - \omega t)}\, \hat{x}$$

$$\tilde{\vec{B}}_R(z,t) = -\frac{E_{0R}}{v_1}\, e^{i(-k_1 z - \omega t)}\, \hat{y}$$

NOTE "−" SIGN FOR k_1 IN exp.

as well as for $\vec{B}_R$ direction.

* Transmitted wave: (T)

$$\vec{E}_T(z,t) = E_{0T}\, e^{i(k_2 z - \omega t)}\, \hat{x}$$

$$\vec{B}_T(z,t) = \frac{E_{0T}}{v_2}\, e^{i(k_2 z - \omega t)}\, \hat{y}$$

$\omega = k_1 v_1 = k_2 v_2$ = const. across boundary

In the class we stopped here.
We will continue derivations below on April 7th

BOUNDARY CONDITIONS
FOR $z=0$

$$E_{0I} + E_{0R} = E_{0T} \qquad \text{(i)}$$

$$\frac{1}{\mu_1}\left(\frac{1}{v_1} \cdot E_{0I} - \frac{1}{v_1} E_{0R}\right) = \frac{1}{\mu_2}\left(\frac{1}{v_2} E_{0T}\right) \qquad \text{(ii)}$$

(iii) and (iv) are not useful since $E^{\parallel} = B^{\parallel} = 0$ (normal incidence)

(ii) rewritten:

$$E_{0I} - E_{0R} = \frac{\mu_1 v_1}{\mu_2 v_2} E_{0T} \qquad \text{(ii)}$$

*Note: If we multiply (i) by (ii):

$$E_{0I}^2 - E_{0R}^2 = \frac{\mu_1 v_1}{\mu_2 v_2} E_{0T}^2 \quad , \text{ OR:}$$

$$\frac{1}{\mu_1 v_1} E_{0I}^2 = \frac{1}{\mu_1 v_1} E_{0R}^2 + \frac{1}{\mu_2 v_2} E_{0T}^2$$

$$\underset{\substack{\uparrow \\ \text{Incident Intensity}}}{\frac{1}{2}\varepsilon_1 v_1 E_{0I}^2} = \underset{\substack{\uparrow \\ \text{REFLECTED}}}{\frac{1}{2}\varepsilon_1 v_1 E_{0R}^2} + \underset{\substack{\uparrow \\ \text{TRANSM.}}}{\frac{1}{2}\varepsilon_2 v_2 E_{0T}^2}$$

ENERGY IS CONSERVED

* IF WE ADD (i) + (ii);

and use

$$\beta = \frac{\mu_1 v_1}{\mu_2 v_2} = \frac{\mu_1 n_2}{\mu_2 n_1}$$

$$E_{0T} = \frac{2}{1+\beta} E_{0I}$$

$$E_{0R} = \frac{1-\beta}{1+\beta} E_{0I}$$

If: $\beta < 1 \qquad E_{0R} > 0$,

REFL. is IN PHASE w/ Incident

$\beta > 1 \qquad E_{0R} < 0$

REFL. WAVE is 180° out of phase

Since $\mu \sim \mu_0 \quad \beta \simeq \frac{v_1}{v_2} = \frac{n_2}{n_1}$

$$E_{0T} \cong \left|\frac{2n_1}{n_1+n_2}\right| E_{0I}$$

$$E \sim \left|\frac{n_1 - n_2}{\ldots}\right| E$$

$$E_{0R} \cong \left| \frac{n_1 - n_2}{n_1 + n_2} \right| E_{0I}$$

Reflection coefficient:

$$R \equiv \frac{I_R}{I_I} = \left(\frac{E_{0R}}{E_{0I}} \right)^2 = \left(\frac{n_1 - n_2}{n_1 + n_2} \right)^2$$

$$T \equiv \frac{I_T}{I_I} = \left(\frac{E_{0T}}{E_{0I}} \right)^2 \cdot \frac{\varepsilon_2 v_2}{\varepsilon_1 v_1} \simeq \frac{4 n_1 n_2}{(n_1 + n_2)^2}$$

* NOTE THAT $R + T = 1$