PHYS 100C, LECTURE 3

Sunday, April 05, 2009
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FROM Lecture #2:

* FOR NORMAL INCIDENCE,

INCIDENT (I) WAVE:

$$\vec{E}_I(z,t) = E_{0I}\, e^{i(k_1 z - \omega t)}\,\hat{x}$$

$$\vec{B}_I(z,t) = \frac{E_{0I}}{c}\, e^{i(k_1 z - \omega t)}\,\hat{y}$$

where $v_1 = \frac{1}{\sqrt{\varepsilon_1 \mu_1}}$, $\quad k_1 = \frac{\omega}{v_1}$

* Reflected WAVE: (R)

$$\vec{E}_R(z,t) = E_{0R}\, e^{i(-k_1 z - \omega t)}\,\hat{x}$$

$$\vec{B}_R(z,t) = -\frac{E_{0R}}{v_1}\, e^{i(-k_1 z - \omega t)}\,\hat{y}$$

Note "−" sign for k_1 in exp. as well as for $\vec{B}_R$ direction.

* Transmitted WAVE: (T)

$$\vec{E}_T(z,t) = E_{0T}\, e^{i(k_2 z - \omega t)}\,\hat{x}$$

$$\vec{B}_T(z,t) = \frac{E_{0T}}{v_2}\, e^{i(k_2 z - \omega t)}\,\hat{y}$$

BOUNDARY CONDITIONS
FOR $z=0$

$$E_{0I} + E_{0R} = E_{0T} \qquad (iii)$$

$$\frac{1}{\mu_1}\left(\frac{1}{v_1} \cdot E_{0I} - \frac{1}{v_1} E_{0R}\right) = \frac{1}{\mu_2}\left(\frac{1}{v_2} E_{0T}\right) \quad (iv)$$

(i) AND (ii) are NOT USEFUL
SINCE $E^{\perp} = B^{\perp} = 0$ (NORMAL INCIDENCE)

(iv) REWRITTEN:

$$E_{0I} - E_{0R} = \frac{\mu_1 v_1}{\mu_2 v_2} E_{0T} \qquad (iv)$$

*NOTE: IF WE MULTIPLY (iii) by (iv)

$$E_{0I}^2 - E_{0R}^2 = \frac{\mu_1 v_1}{\mu_2 v_2} E_{0T}^2 \quad , \text{ OR:}$$

$$\frac{1}{\mu_1 v_1} E_{0I}^2 = \frac{1}{\mu_1 v_1} E_{0R}^2 + \frac{1}{\mu_2 v_2} E_{0T}^2$$

$$\underbrace{\frac{1}{2} \varepsilon_1 v_1 E_{0I}^2}_{\text{Incident Intensity}} = \underbrace{\frac{1}{2} \varepsilon_1 v_1 E_{0R}^2}_{\text{REFLECTED}} + \underbrace{\frac{1}{2} \varepsilon_2 v_2 E_{0T}^2}_{\text{TRANSM.}}$$

ENERGY IS CONSERVED

* IF WE ADD (i)+(ii);
and use $\beta = \frac{\mu_1 v_1}{\mu_2 v_2} = \frac{\mu_1 n_2}{\mu_2 n_1}$

$$E_{0T} = \frac{2}{1+\beta} E_{0I}$$

$$E_{0R} = \frac{1-\beta}{1+\beta} E_{0I}$$

If: $\beta < 1$ $\quad E_{0R} > 0$,

REFL. is IN PHASE w/ Incident

$\beta > 1$ $\quad E_{0R} < 0$

REFL. WAVE is 180° out of phase

Since $\mu \sim \mu_0$ $\quad \beta \approx \frac{v_1}{v_2} = \frac{n_2}{n_1}$

$$E_{0T} \cong \left|\frac{2n_1}{n_1+n_2}\right| E_{0I}$$

$$E_{0R} \cong \left|\frac{n_1-n_2}{n_1+n_2}\right| E_{0I}$$

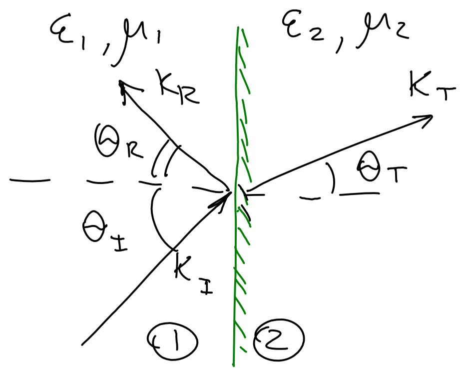
Reflection coefficient:

$$R \equiv \frac{I_R}{I_I} = \left(\frac{E_{0R}}{E_{0I}}\right)^2 = \left(\frac{n_1-n_2}{n_1+n_2}\right)^2$$

$$T \equiv \frac{I_T}{I_I} = \left(\frac{E_{0T}}{E_{0I}}\right)^2 \cdot \frac{\varepsilon_2 v_2}{\varepsilon_1 v_1} \simeq \frac{4n_1 n_2}{(n_1+n_2)^2}$$

* NOTE THAT $R + T = 1$

* Oblique Incidence case:



Incident wave:

$$\tilde{E}_I(\vec{r},t) = \tilde{E}_{0I}\, e^{i(\vec{k}_I\cdot\vec{r}-\omega_I t)};\quad \vec{B}_I = \frac{(\vec{k}_I\times\vec{E}_I)}{v_1}$$

Reflected:

$$\tilde{E}_R(\vec{r},t) = \tilde{E}_{0R}\, e^{i(\vec{k}_R\cdot\vec{r}-\omega_R t)};\quad B_R = \frac{(\vec{k}_R\times\vec{E}_R)}{v_1}$$

Transmitted:

$$\tilde{E}_T(\vec{r},t) = \vec{E}_{0T}\cdot e^{i(\vec{k}_T\cdot\vec{r}-\omega_T t)};\quad B_T = \frac{(\vec{k}_T\times\vec{E}_T)}{v_2}$$

where $k_I = \frac{\omega_I}{v_1}$; etc.

Boundary conditions: $(z=0)$

$$(\,)\cdot e^{i(\vec{k}_I\cdot\vec{r}-\omega_I t)} + (\,)\cdot e^{i(\vec{k}_R\cdot\vec{r}-\omega_R t)} = (\,)\cdot e^{i(\vec{k}_T\cdot\vec{r}-\omega_T t)}$$

As Prob. 9.15 shows, this means:

$\omega_I = \omega_R = \omega_T$ (frequencies are the same)

(We will use ω)

and

$$\vec{k}_I\cdot\vec{r} = \vec{k}_R\cdot\vec{r} = \vec{k}_T\cdot\vec{r}$$

$$(k_I)_x\cdot x + (k_I)_y\cdot y = (k_R)_x\cdot x + (k_R)_y\cdot y = (k_T)_x\cdot x + (k_T)_y\cdot y$$

($z=0$ so it doesn't enter)

For $x=0$:

$$(k_I)_y = (k_R)_y = (k_T)_y$$

For $y=0$

$$(k_I)_x = (k_R)_x = (k_T)_x$$

Let's define $\hat{x}$ axis so that k_I is in xz plane (of incidence)

or $(k_I)_y = 0 \;(= (k_R)_y = (k_T)_y)$

1st Law: $\vec{k}_I, \vec{k}_R, \vec{k}_T$ form a "plane of incidence", which also includes SURFACE normal

For x-components:

$$k_I \cdot \sin\theta_I = k_R \cdot \sin\theta_R = k_T \cdot \sin\theta_T$$

Since $k_I = k_R = \frac{\omega}{v_1}$ $\quad \theta_I = \theta_R$

2nd Law: ANGLE OF incidence is equal to the angle of reflection

$$\theta_I = \theta_R$$

(LAW OF REFLECTION)

FOR THIRD COMPONENT, $k_T = \frac{\omega}{v_2}$ and $k_I = \frac{\omega}{v_1}$:

$$\frac{\omega}{v_1} \cdot \sin\theta_I = \frac{\omega}{v_2} \cdot \sin\theta_T$$

$$\frac{\sin\theta_T}{\sin\theta_I} = \frac{v_2}{v_1} = \frac{n_1}{n_2}$$

3rd Law: Snell's Law or LAW OF REFRACTION:

$$\frac{\sin\theta_T}{\sin\theta_I} = \frac{n_1}{n_2}$$

* **BOUNDARY CONDITIONS:**

(exponents cancel out)

(i) $\varepsilon_1(\tilde{E}_{0I}+\tilde{E}_{0R})_z=\varepsilon_2(\tilde{E}_{0T})_z$

(ii) $(\tilde{B}_{0I}+\tilde{B}_{0R})_z=(\tilde{B}_{0T})_z$

(iii) $(\tilde{E}_{0I}+\tilde{E}_{0R})_{x,y}=(\tilde{E}_{0T})_{x,y}$

(iv) $\frac{1}{\mu_1}(\tilde{B}_{0I}+\tilde{B}_{0R})_{x,y}=\frac{1}{\mu_2}(\tilde{B}_{0T})_{x,y}$

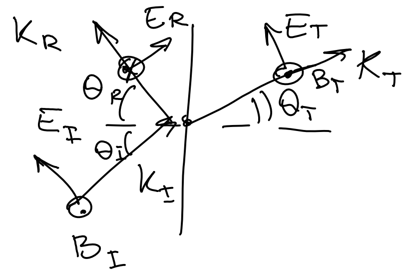
and $\bar{B}=\frac{1}{v}(\bar{k}\times\bar{E})$

For p-polalized wave
($\vec{E}$ ∥ plane of incidence):

(i)$\Rightarrow$ $\varepsilon_1(-\tilde{E}_{0I}\cdot\sin\theta_I+\tilde{E}_{0R}\cdot\sin\theta_R)=-\varepsilon_2 E_{0T}\sin\theta_T$

$B\perp z$, so (ii): $0=0$

(iii) $\tilde{E}_{0I}\cdot\cos\theta_I+\tilde{E}_{0R}\cdot\cos\theta_R=\tilde{E}_{0T}\cdot\cos\theta_T$



(iv)$\Rightarrow$ $\frac{1}{\mu_1 v_1}(\tilde{E}_{0I}-\tilde{E}_{0R})=\frac{1}{\mu_2 v_2}\tilde{E}_{0T}$

since $\theta_I=\theta_R$, (i) & (iv):

$$\tilde{E}_{0I}-\tilde{E}_{0R}=\tilde{E}_{0T}\cdot\beta$$

where $\beta\equiv\frac{\mu_1 v_1}{\mu_2 v_2}=\frac{\varepsilon_2}{\varepsilon_1}\cdot\frac{\sin\theta_T}{\sin\theta_I}=\frac{\mu_1 n_2}{\mu_2 n_1}$

(iii) $\Rightarrow \tilde{E}_{0I} + \tilde{E}_{0R} = \tilde{E}_{0T} \cdot \alpha$

$$\alpha \equiv \frac{\cos\theta_T}{\cos\theta_I}$$

add / subtract: **Fresnel Eqs:**

$$\boxed{\tilde{E}_{0R} = \left(\frac{\alpha - \beta}{\alpha + \beta}\right)\tilde{E}_{0I} \; ; \; \tilde{E}_{0T} = \left(\frac{2}{\alpha + \beta}\right)\tilde{E}_{0I}}$$

since $\cos\theta_T = \sqrt{1 - \sin^2\theta_T} = \sqrt{1 - \frac{n_1^2}{n_2^2}\sin^2\theta_I}$

$$\alpha(\theta_I) = \sqrt{1 - \frac{n_1^2 \sin^2\theta_I}{n_2^2}} \cdot \frac{1}{\cos\theta_I}$$

$$R \equiv \frac{I_R}{I_I} = \left(\frac{E_{0R}}{E_{0I}}\right)^2 = \left(\frac{\alpha - \beta}{\alpha + \beta}\right)^2$$

$$T \equiv \frac{I_T}{I_I} = \frac{\varepsilon_2 v_2}{\varepsilon_1 v_1} \cdot \left(\frac{E_{0T}}{E_{0I}}\right)^2 \frac{\cos\theta_T}{\cos\theta_I} = \alpha\beta\left(\frac{2}{\alpha + \beta}\right)^2$$

* Brewster Angle:

when $\alpha = \beta$ $\quad R = 0$

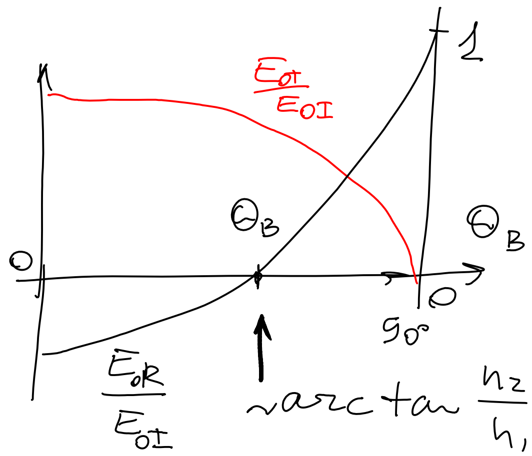
$$\alpha = \beta \Rightarrow \quad 1 - \left(\frac{n_1}{n_2}\right)^2 \cdot \sin^2\theta_B = \beta^2 \cdot \cos^2\theta_B = \beta^2(1 - \sin^2\theta_B)$$

$$\sin^2\theta_B = \frac{1 - \beta^2}{(n_1/n_2)^2 - \beta^2} \qquad \text{since } \beta \approx \frac{n_2}{n_1}$$

$$\sin^2\theta_B = \frac{1 - \beta^2}{\left(\frac{1}{\beta}\right)^2 - \beta^2} = \beta^2 \frac{1 - \beta^2}{1 - \beta^4} = \frac{\beta^2}{1 + \beta^2}$$

OR $\quad \tan^2\theta_B = \dfrac{\sin^2\theta_B}{1 - \sin^2\theta_B} = \beta^2$

$$\tan\Theta_B = \frac{n_2}{n_1}$$



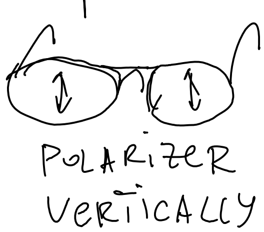
E_{0R} is < 0 (out of phase by 180° w.r. to E_{0I})

* Application #1: polarized SUNGLASSES

p-polarized waves ⇒ reflection is suppresed around Θ_B ($R = 0$ at Θ_B)

s-polarized waves "survive" reflection

Result: reflected sunlight is preferrentially s-polarized ($E \perp$ plane of incidence)



* APPLICATION #2

Brewster ANGLE MICROSCOPY: