PHYS 100C, Homework #5, Due Thursday, May 7th, 8AM (in class)

Problem 10.13 A particle of charge q moves in a circle of radius a at constant angular velocity ω. (Assume that the circle lies in the xy plane, centered at the origin, and at time $t = 0$ the charge is at $(a, 0)$, on the positive x axis.) Find the Liénard-Wiechert potentials for points on the z axis.

Problem 10.17 ~~Derive Eq. 10.63. First~~ show that

$$\frac{\partial t_r}{\partial t} = \frac{\mathscr{r} c}{\boldsymbol{\mathscr{r}} \cdot \mathbf{u}}.$$

Problem 10.20 For the configuration in Prob. 10.13, find the electric and magnetic fields at the center. From your formula for **B**, determine the magnetic field at the center of a circular loop carrying a steady current I, and compare your answer with the result of Ex. 5.6

Problem 11.1 Check that the retarded potentials of an oscillating dipole (Eqs. 11.12 and 11.17) satisfy the Lorentz gauge condition. Do *not* use approximation 3.

PHYS 100C HW#5 Solutions

Tuesday, May 12, 2009
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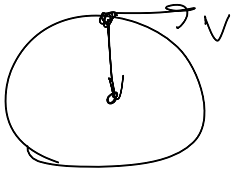
***10.13** Since $\vec{v} \perp \vec{\mathscr{r}}$



$(\vec{\mathscr{r}} = z \cdot \hat{z} + a \cdot \hat{p}$

where $\hat{p}$ is direction toward center, $\vec{v} \cdot \vec{\mathscr{r}} = z \cdot \underbrace{\vec{v} \cdot \hat{z}}_{0} + a \underbrace{\vec{v} \cdot \vec{p}}_{0})$

$$V(r,t) = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{\sqrt{z^2+a^2}}$$

$$\bar{A}(r,t) = \frac{\bar{v}}{c^2} \cdot V(r,t)$$

If position of particle (retarded)

$x = a \cdot \cos \omega t_r$

$y = a \cdot \sin \omega t_r$

where $t_r = t - \frac{\sqrt{z^2+a^2}}{c}$

then velocity (retarded)

$\vec{v} = \omega a(-\sin \omega t_r \cdot \hat{x} + \cos \omega t_r \cdot \hat{y})$

$$\bar{A} = \frac{q \omega a(-\sin \omega t \cdot \hat{x} + \cos \omega t_r \cdot \hat{y})}{4\pi\varepsilon_0 c^2 \sqrt{z^2+a^2}}$$

***10.17** $c(t - t_r) = |\mathscr{r}|$

$\frac{\partial}{\partial t}$ both sides:

$$c \frac{\partial}{\partial t}(t - t_r) = \frac{\partial |\mathscr{r}|}{\partial t} = \frac{\partial \sqrt{\bar{\mathscr{r}}^2}}{\partial t}$$

to get vector $\bar{r}$

$$c\left(1 - \frac{\partial t_r}{\partial t}\right) = \frac{1}{2\sqrt{\bar{\mathscr{r}}^2}} \cdot \frac{\partial(\bar{\mathscr{r}}^2)}{\partial t} = \frac{2\bar{\mathscr{r}}}{2|\mathscr{r}|} \cdot \frac{\partial \bar{\mathscr{r}}}{\partial t}$$

$$c\left(1-\frac{\partial t_r}{\partial t}\right)=\frac{1}{2\sqrt{\bar{\mathscr{r}}^2}}\cdot\frac{\partial(\bar{\mathscr{r}}^2)}{\partial t}=\underbrace{\frac{2\bar{\mathscr{r}}}{2|\mathscr{r}|}}_{\hat{\mathscr{r}}}\cdot\frac{\partial\bar{\mathscr{r}}}{\partial t}$$

Since $\bar{\mathscr{r}}=\bar{r}-\bar{\omega}(t)$

$$\frac{\partial\bar{\mathscr{r}}}{\partial t}=\underbrace{\frac{\partial\bar{r}}{\partial t}}_{0}-\frac{\partial\vec{\omega}(t)}{\partial t}=-\frac{\partial\vec{\omega}}{\partial t_R}\cdot\frac{\partial t_R}{\partial t}=-\vec{v}\cdot\frac{\partial t_R}{\partial t}$$

Since $\frac{\partial\bar{\omega}}{\partial t_R}=\vec{v}$ (retarded)

$$c\left(1-\frac{\partial t_r}{\partial t}\right)=\hat{\mathscr{r}}\cdot\left(-\vec{v}\cdot\frac{\partial t_R}{\partial t}\right)$$

$$\frac{\partial t_r}{\partial t}=\frac{c}{c-\hat{\mathscr{r}}\cdot\vec{v}}=\frac{c|\mathscr{r}|}{c|\mathscr{r}|-\bar{\mathscr{r}}\cdot v}=\frac{c\mathscr{r}}{\bar{\mathscr{r}}\cdot\bar{u}}$$

10.20 $u=c\hat{\mathscr{r}}-\vec{v}$

Remember that $\vec{\mathscr{r}}\perp\vec{v}$, $\vec{a}\parallel\vec{r}$

$$\mathscr{r}\times(u\times a)=(\bar{\mathscr{r}}\cdot\bar{a})u-(\bar{\mathscr{r}}\cdot\bar{u})\bar{a}$$

$$\bar{\mathscr{r}}\cdot\bar{u}=\bar{\mathscr{r}}\cdot(c\hat{\mathscr{r}}-\bar{v})=c\mathscr{r}$$

$$\bar{\mathscr{r}}\cdot\bar{a}=\omega^2\mathscr{r}^2$$

$$\mathscr{r}\times(u\times a)=\cancel{c\omega^2\mathscr{r}^2\cdot\hat{\mathscr{r}}}-\underbrace{\omega^2\mathscr{r}^2}_{v^2}\cdot\vec{v}-\cancel{c\mathscr{r}\cdot\omega^2\bar{\mathscr{r}}}$$

$$E=\frac{q}{4\pi\varepsilon_0}\cdot\frac{\mathscr{r}}{(\mathscr{r}\cdot u)^3}\left((c^2-v^2)\bar{u}+\bar{\mathscr{r}}\times(\bar{u}\times\bar{a})\right)=$$

$$=\frac{q}{4\pi\varepsilon_0}\cdot\frac{\mathscr{r}}{(c\mathscr{r})^3}\cdot\left((c^2-v^2)\cdot c\hat{\mathscr{r}}-c^2\bar{v}+\cancel{v^2\bar{v}}-\cancel{v^2\bar{v}}\right)=$$

$$=\frac{q}{4\pi\varepsilon_0 c^2\mathscr{r}^2}\Big[(c^2-\omega^2\mathscr{r}^2)(-\cos\omega t_R\cdot\hat{x}-\sin\omega t_R\cdot\hat{y})-$$
$$-c\cdot\omega\mathscr{r}(-\sin\omega t_R\cdot\hat{x}+\cos\omega t_R\cdot\hat{y})\Big]$$

We used $\hat{r} = -\frac{\bar{\omega}}{|\omega|} = -(\cos\omega t_R \cdot \hat{x} + \sin\omega t_R \cdot \hat{y})$

and $\bar{v} = \omega r(-\sin\omega t_R \cdot \hat{x} + \cos\omega t_R \cdot \hat{y})$

$$B = \frac{1}{c}(\hat{r} \times \vec{E})$$

$$\hat{r} \times (c^2 - v^2) \cdot c\hat{r} = 0$$

$$\hat{r} \times (-c^2 \bar{v}) = -c^2(\hat{r} \times \bar{v}) = c^2 \omega r \cdot \hat{z}$$

$$B = \frac{1}{c} \cdot \frac{q}{4\pi\varepsilon_0} \cdot \frac{r}{(cr)^3} \cdot c^2 \omega r \cdot \hat{z} =$$

$$= \frac{q}{4\pi\varepsilon_0} \cdot \frac{\omega}{c^2 r} \cdot \hat{z}$$

*11.1

$$A = -\underbrace{\frac{\mu_0 p_0 \omega}{4\pi r}}_{K} \cdot \sin u \cdot (\cos\theta \cdot \hat{r} - \sin\theta \cdot \hat{\theta})$$

where $u = \omega(t - r/c)$

$$\nabla \cdot A = K\left(\frac{1}{r^2} \cdot \frac{\partial}{\partial r}\left(r^2 \cdot \frac{\sin u}{r} \cdot \cos\theta\right) + \frac{1}{r \cdot \sin\theta} \times\right.$$

$$\left. \times \frac{\partial}{\partial\theta}\left(-\sin^2\theta \frac{1}{r} \cdot \sin u\right)\right) =$$

$$= K\left[\left(\frac{\sin u}{r^2} - \frac{\omega}{rc}\cos u\right)\cos\theta - \frac{2\sin\theta \cdot \cos\theta}{r^2 \sin\theta}\sin u\right] =$$

$$= K\left(-\frac{\sin u}{r^2} + \frac{\omega}{rc}\cos u\right)\cos\theta$$

$$\frac{\partial V}{\partial t} = \frac{p_0 \cos\theta}{4\pi\varepsilon_0 r}\left(\cos u - \frac{\omega}{r}\sin u\right) =$$

$$= -\frac{p_0 \omega}{4\pi\varepsilon_0} \cdot \left(\frac{\sin u}{r^2} + \frac{\omega}{rc} \cos u \right) \cos\theta$$

$$\nabla \cdot A = -\mu_0 \varepsilon_0 \cdot \frac{\partial V}{\partial t}$$