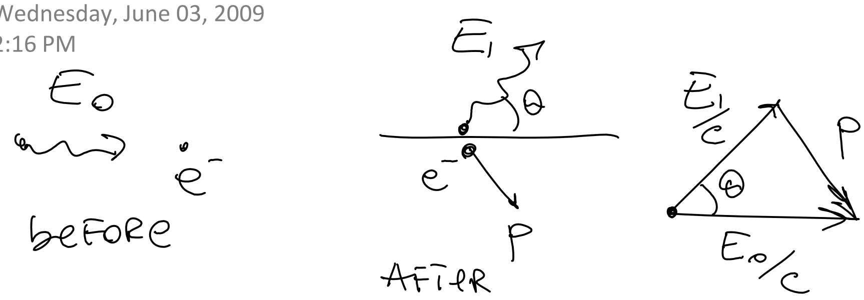
Homework #7

Wednesday, June 03, 2009
2:16 PM

X:



"No free lunch" momentum:

$$p^2 = \left(\frac{E_1}{c}\right)^2 + \left(\frac{E_0}{c}\right)^2 - 2\frac{E_1 E_0}{c^2}\cdot\cos\theta$$

"No free lunch" energy:

$$E_0 + mc^2 = E_1 + \sqrt{m^2c^4 + p^2c^2}$$

$$\left(E_0 - E_1 + mc^2\right)^2 = m^2c^4 + p^2c^2$$

$$\underline{E_0^2 + E_1^2} - 2E_0E_1 + \underline{m^2c^4} + 2(E_0 - E_1)mc^2 =$$

$$= m^2c^4 + \underline{p^2c^2}$$

From momentum eq: $\times c^2$

$$\underline{E_0^2 + E_1^2} - 2E_0E_1\cos\theta = \underline{p^2c^2}$$

subtract the two:

$$2(E_0 - E_1)mc^2 = 2E_0E_1(1 - \cos\theta)$$

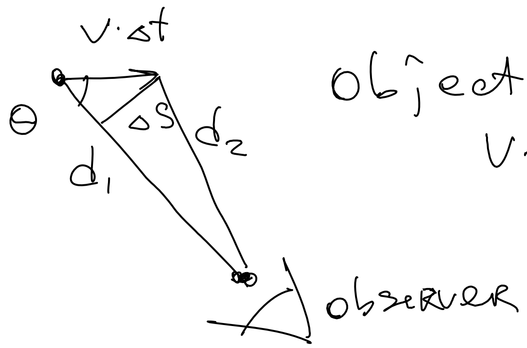
$$\frac{1}{E_1} - \frac{1}{E_0} = \frac{1 - \cos\theta}{mc^2}$$

$$E = h\nu = h\frac{c}{\lambda} \quad \Rightarrow \quad \frac{1}{E} = \frac{\lambda}{hc}$$

$$\frac{\lambda_1}{hc} - \frac{\lambda_0}{hc} = \frac{1 - \cos\theta}{mc^2}$$

$$\lambda_1 - \lambda_0 = \frac{h}{mc}(1 - \cos\theta)$$

12.6



Object travels
$v \cdot \Delta t$

observer "observes" motion

$\Delta S = v \cdot \Delta t \cdot \sin\theta$

time between two photons is

$\Delta t' = \Delta t - \frac{d_1 - d_2}{c} = \Delta t - \frac{v \cdot \Delta t \cdot \cos\theta}{c}$

Observed velocity is

$u = \frac{\Delta S}{\Delta t'} = \frac{v \cdot \Delta t \cdot \sin\theta}{\Delta t - \frac{v \cdot \Delta t \cdot \cos\theta}{c}} = \frac{v \cdot \sin\theta}{1 - \frac{v\cos\theta}{c}}$

Could easily be $> c$

12.7 In muon's F.O.R. lifetime is γt

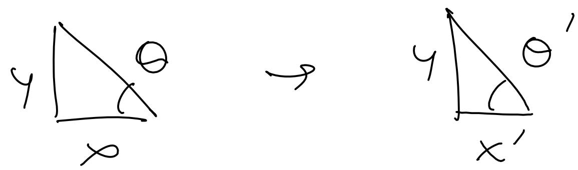
$v = \frac{d}{\gamma t} = \frac{d}{t}\sqrt{1 - \left(\frac{v}{c}\right)^2}$

$\left(\frac{v}{c}\right)^2 = \left(\frac{d}{ct}\right)^2 \cdot \left(1 - \left(\frac{v}{c}\right)^2\right)$

$$\left(\frac{v}{c}\right)^2 = \frac{1}{1+\left(\frac{ct}{d}\right)^2}$$

$$V = \frac{c}{\sqrt{1+\left(\frac{ct}{d}\right)^2}} = 0.8\,c$$

12.10



$$x' = \gamma \cdot x$$

$$\tan\theta' = \frac{y}{x'} = \frac{y}{\gamma x} = \frac{\tan\theta}{\gamma}$$

$$\theta' = \arctan\left(\sqrt{1-v^2/c^2}\cdot \tan\theta\right)$$

12.17

$$\bar{a}^0 = \gamma(a^0 - \beta a^1)$$
$$\bar{a}^1 = \gamma(a^1 - \beta a^0)$$
$$\bar{a}^2 = a^2$$
$$\bar{a}^3 = a^3$$

$$-\bar{a}^0\bar{b}^0 + \bar{a}^1\bar{b}^1 + \bar{a}^2\bar{b}^2 + \bar{a}^3\bar{b}^3 =$$

$$= -\gamma^2(a^0-\beta a^1)(b^0-\beta b^1) +$$

$$+\gamma^2(a^1-\beta a^0)(b^1-\beta b^0) + a^2b^2 +$$

$$+ a^3b^3 = -\gamma^2\big(a^0b^0 - \beta a^1b^0 - \beta a^0 b^1 +$$

$$+\beta^2 a^1b^1 - a^1b^1 + \beta a^0 b^1 + \beta a^1 b^0 - \beta^2 a^0 b^0\big) +$$

$$+ a^2b^2 + a^3b^3 = -\gamma^2(a^0b^0 - a^1b^1)\underbrace{(1-\beta^2)}_{1/\gamma^2} +$$

$$+ a^2b^2 + a^3b^3 = -a^0b^0 + a^1b^1 + a^2b^2 + a^3b^3$$

12.29 $\gamma mc^2 - mc^2 = n \cdot mc^2$

$$n = \gamma - 1$$

$$\left(1 - \frac{v^2}{c^2}\right)^{1/2} = \frac{1}{\gamma} = \frac{1}{n+1}$$

$$\frac{v^2}{c^2} = 1 - \left(\frac{1}{n+1}\right)^2$$

$$v = c \cdot \left(1 - \left(\frac{1}{n+1}\right)^2\right)^{1/2} \quad \text{OR}$$

$$v = c \cdot \frac{\sqrt{n(n+2)}}{n+1}$$

12.30

$$\tilde{p}_n = \gamma\left(p_n - \beta \frac{E_n}{c}\right)$$

$$\sum \tilde{p}_n = \gamma\left(\sum p_n - \frac{\beta}{c} \sum E_n\right) = 0$$

(C.O.M. FRAME)

$$\beta = c \cdot \frac{\sum p_n}{\sum E_n}$$

$$v = c^2 \frac{\sum p_n}{\sum E_n}$$