Problem 9.3

Wednesday, April 01, 2009
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$$A_3 e^{i\delta_3} = A_1 e^{i\delta_1} + A_2 e^{i\delta_2}$$

Multiply both sides by complex conjugates (since $|\tilde{a}|^2 = \tilde{a} \cdot \tilde{a}^*$)

$$A_3^2 = (A_1 e^{i\delta_1} + A_2 e^{i\delta_2})(A_1 e^{-i\delta_1} + A_2 e^{-i\delta_2})$$

$$A_3^2 = A_1^2 + A_2^2 + A_1 A_2 \left(e^{i(\delta_2-\delta_1)} + e^{i(\delta_1-\delta_2)}\right)$$

Since $e^{ix} = \cos x + i \sin x$

$$e^{ix} + e^{-ix} = 2\cos x$$

$$\boxed{A_3^2 = A_1^2 + A_2^2 + 2A_1 \cdot A_2 \cdot \cos(\delta_2 - \delta_1)}$$

Equating Re parts:

$$A_3 \cdot \cos\delta_3 = A_1 \cdot \cos\delta_1 + A_2 \cdot \cos\delta_2 \qquad (1)$$

Im. parts:

$$A_3 \cdot \sin\delta_3 = A_1 \sin\delta_1 + A_2 \cdot \sin\delta_2 \qquad (2)$$

Divide (2) by (1) (A_3 drops out):

$$\boxed{\tan\delta_3 = \frac{A_1 \cdot \sin\delta_1 + A_2 \cdot \sin\delta_2}{A_1 \cdot \cos\delta_1 + A_2 \cdot \cos\delta_2}}$$

*** Answer:**

$$A_3 = \sqrt{A_1^2 + A_2^2 + 2A_1 \cdot A_2 \cdot \cos(\delta_2 - \delta_1)}$$

$$\delta_3 = \arctan\left(\frac{A_1 \cdot \sin\delta_1 + A_2 \cdot \sin\delta_2}{A_1 \cdot \cos\delta_1 + A_2 \cdot \cos\delta_2}\right)$$

Problem 9.3, cont'd:

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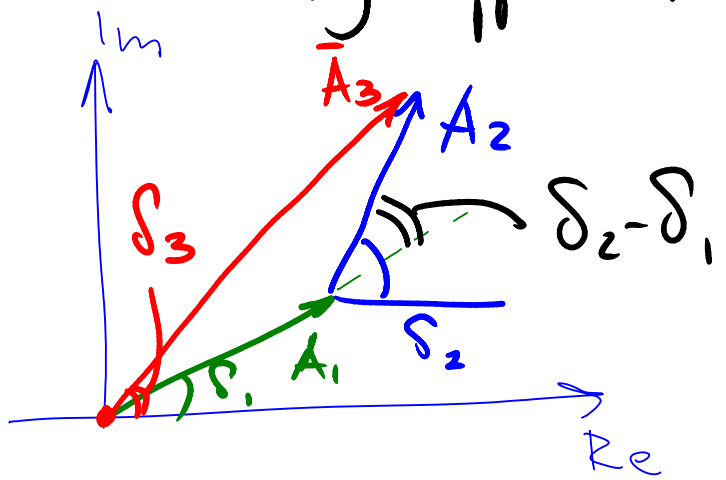
(Alternative solution)

\+ Geometry approach:



From A_1, A_2, A_3 triangle:

$$A_3^2 = A_1^2 + A_2^2 + 2A_1 \cdot A_2 \cos(\delta_2 - \delta_1)$$

$$\tan \delta_3 = \frac{A_2 \cdot \sin \delta_2 + A_1 \cdot \sin \delta_1}{A_2 \cdot \cos \delta_2 + A_1 \cdot \cos \delta_2}$$

(Projections of A_1, A_2 on Re and im axis).

Problem 9.5

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Boundary Conditions at $z=0$:

$$g_I(-v_1t) + h_R(v_1t) = g_T(-v_2t) \qquad (1)$$

$$\frac{\partial g_I}{\partial z}(-v_1t) + \frac{\partial h_R}{\partial z}(v_1t) = \frac{\partial g_T}{\partial z}(-v_2t) \qquad (2)$$

Need to convert $\frac{\partial}{\partial z}$ to $\frac{\partial}{\partial t}$
so we can integrate over t
(cannot integrate over z since $z=0$).

Note that $\frac{\partial g}{\partial z} = \frac{\partial g}{\partial u} \cdot \frac{\partial u}{\partial z}$

and $\frac{\partial g}{\partial t} = \frac{\partial g}{\partial u} \cdot \frac{\partial u}{\partial t}$

where g is g_I, h_R or g_T
and u is $z-v_1t$; $z+v_1t$ or $z-v_2t$
respectively

therefore $\frac{\partial g}{\partial z} = \frac{\partial g}{\partial t} \cdot \frac{\partial u}{\partial z} \cdot \left(\frac{\partial u}{\partial t}\right)^{-1}$

$\frac{\partial u}{\partial z} = 1$ for all functions

$\frac{\partial u}{\partial t}$ is $-v_1$; $+v_1$ and $-v_2$

therefore Eq. (2) becomes:

$$\frac{\partial g_I(-v_1t)}{\partial t} \cdot \left(-\frac{1}{v_1}\right) + \frac{\partial h_R(v_1t)}{\partial t} \cdot \frac{1}{v_1} = \frac{\partial g_T(-v_2t)}{\partial t} \cdot \left(-\frac{1}{v_2}\right) \qquad (3)$$

Multiply $\times(-v_1)$, and integrating $\int dt$:

$$g_I(-v_1t) - h_R(v_1t) = \frac{v_1}{v_2} g_T(-v_2t) + \text{const.} \qquad (4)$$

Add (1)+(4) $\Rightarrow h_R$ drops out:

$$g_T(-v_2 t) = \frac{2v_2}{v_1+v_2}\, g_I(-v_1 t) + \text{const.}$$

OR, since g_T is a function of $z - v_2 t$

$$\boxed{g_T(z - v_2 t) = \frac{2V_2}{V_1+V_2}\, g_I\left(\frac{v_1}{v_2} z - v_1 t\right) + C}$$

(Generally,

$$g_T(x) = \frac{2v_2}{v_1+v_2}\, g_I\left(\frac{v_1}{v_2} x\right) + \text{const.}\Big)$$

h_R can be calculated by adding

$$(1) - \frac{V_2}{V_1} \cdot (4):$$

$$g_I(-v_1 t) \cdot \frac{v_1 - v_2}{v_1} + h_R(v_1 t) \cdot \frac{v_1+v_2}{v_1} = 0$$

$$h_R(v_1 t) = \frac{v_2 - v_1}{v_2+v_1} \cdot g_I(-v_1 t) + \text{const.}$$

$$h_R(x) = \frac{v_2 - v_1}{v_2+v_1} \cdot g_I(-x) + \text{const.}$$

OR, as $h_R(z + v_1 t)$:

$$\boxed{h_R(z + v_1 t) = \frac{v_2 - v_1}{v_2 + v_1} \cdot g_I(-z - v_1 t) + C}$$

C is the same constant (e.g. (1) for $t=0$)

Problem 9.9

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a) $\vec{E} = E_0 \cdot \cos(|\vec{k}|x + \omega t)\,\hat{z}$

$\hat{k}$ along $-\hat{x}$ $\qquad$ Polarization $\hat{n} \parallel \hat{z}$

$\vec{k} = -\frac{\omega}{c}\cdot\hat{x} \qquad \hat{n} = \hat{z}$

$$\vec{E} = E_0 \cdot \cos\left(\frac{\omega}{c}x + \omega t\right)\hat{z}$$

$$\vec{B} = \frac{E_0}{c}\cdot\cos\left(\frac{\omega}{c}x + \omega t\right)(\hat{k}\times\hat{n})$$

$$\hat{k}\times\hat{n} = -(\hat{x}\times\hat{z}) = -(-\hat{y}) = \hat{y}$$

$$\vec{B} = \frac{E_0}{c}\cos\left(\frac{\omega}{c}x + \omega t\right)\hat{y}$$

Another check: Poynting vector

$$\vec{S} \upuparrows (\vec{E}\times\vec{B}) \upuparrows (\hat{z}\times\hat{y}) = -\hat{x}$$

b) $\hat{k} = \dfrac{\hat{x}+\hat{y}+\hat{z}}{(1^2+1^2+1^2)^{1/2}} = \dfrac{\hat{x}+\hat{y}+\hat{z}}{\sqrt{3}}$

$k = \frac{\omega}{c}\cdot\hat{k}$ $\qquad$ $\vec{E}$ is in xz plane and $\vec{E}\perp\vec{k}$, or $\vec{E}\cdot\vec{k}=0$

$\hat{n} = a\hat{x} + b\hat{z}$ $\quad$ (xz plane)

$$\hat{n}\cdot\hat{k} = 0 \Rightarrow (a\hat{x}+b\hat{z})(\hat{x}+\hat{y}+\hat{z}) = 0$$

$$a(\hat{x})^2 + b(\hat{z})^2 = 0$$

$$a = -b$$

$$\hat{n} = \frac{\hat{x}-\hat{z}}{\sqrt{1^2+1^2}} = \frac{\hat{x}-\hat{z}}{\sqrt{2}}$$

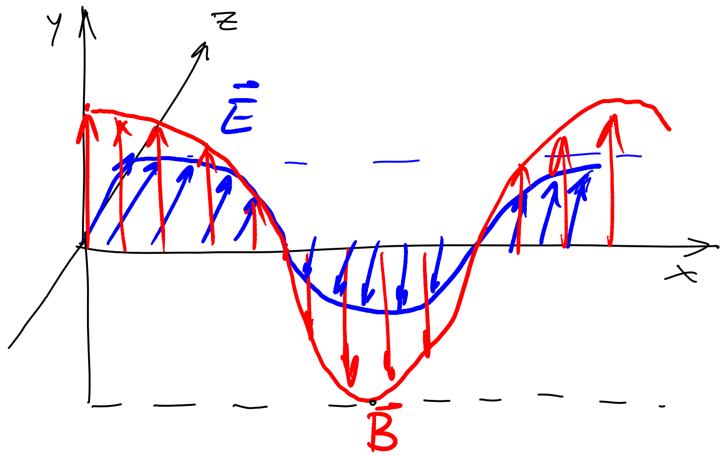
$\vec{B}$ is along $(\hat{k}\times\hat{n})$:

$$(\hat{k}\times\hat{n}) = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 1 & 1 & 1 \\ 1 & 0 & -1 \end{vmatrix}\cdot\frac{1}{\sqrt{3}}\cdot\frac{1}{\sqrt{2}} = \frac{-\hat{x}+\hat{y}+\hat{y}-\hat{z}}{\sqrt{6}}$$

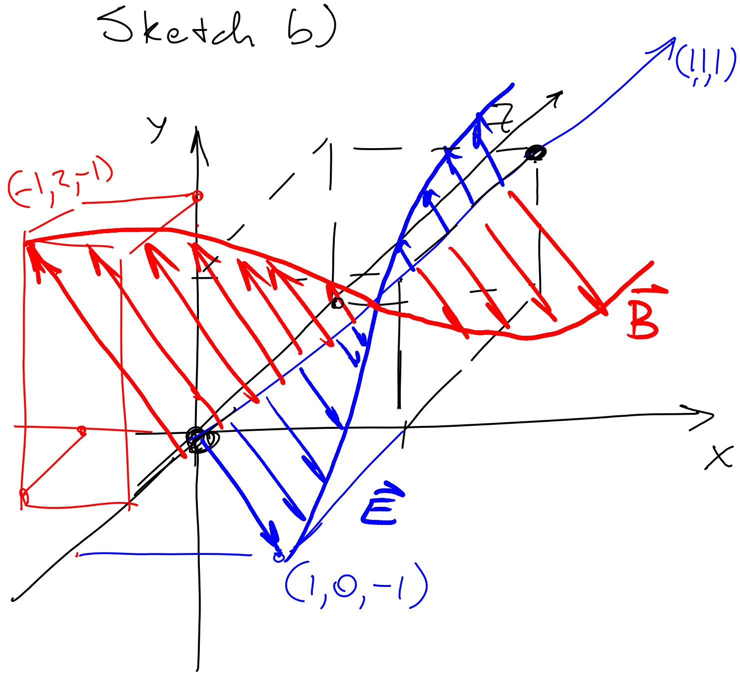
$$\vec{E} = E_0\cdot\cos\left(\frac{\omega}{c}\cdot\frac{x+y+z}{\sqrt{3}} - \omega t\right)\frac{\hat{x}-\hat{z}}{\sqrt{2}}$$

$$\vec{B} = \frac{E_0}{c}\cdot\cos\left(\frac{\omega}{c}\cdot\frac{x+y+z}{\sqrt{3}} - \omega t\right)\frac{-\hat{x}+2\hat{y}-\hat{z}}{\sqrt{6}}$$

Sketch a):



Sketch b)



Problem 9.10

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$$I = 1{,}300 \ \frac{W}{m^2} = \langle S \rangle = \frac{1}{2} \varepsilon_0 c E_0^2$$

$$P_1 = \frac{1}{2} \varepsilon_0 E_0^2 = \frac{I}{c} = \frac{1300 \frac{W}{m^2}}{3 \cdot 10^8 m/c} = 4.33 \cdot 10^{-6} \ Pa$$

Perfect absorber

FOR PERFECT REFLECTOR

$$P_2 = 2 \cdot 4.33 \cdot 10^{-6} \ Pa = 8.66 \cdot 10^{-6} \ Pa$$

$$P_{ATM} = 10^5 \ Pa$$

$$\frac{P_1}{P_{ATM}} = 4.33 \cdot 10^{-11}$$

$$\frac{P_2}{P_{ATM}} = 8.66 \cdot 10^{-11}$$

Problem 9.15

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$$Ae^{iax} + Be^{ibx} = Ce^{icx}$$

For $x=0$

$$\boxed{A+B=C} \quad \text{(easy one!)}$$

Take $\frac{\partial}{\partial x}$ of both sides:

$$iaAe^{iax} + ibBe^{ibx} = icCe^{icx}$$

For $x=0$

$$aA + bB = cC = c(A+B)$$

OR

$$(a-c)A + (b-c)B = 0 \qquad (1)$$

If we took $\frac{\partial}{\partial x}$ 2 times, we get:

$$a^2A + b^2B = c^2C = c^2(A+B)$$

$$(a^2-c^2)A + (b^2-c^2)B = 0 \qquad (2)$$

From (1): $(a-c)A = (c-b)B$

(2): $(a-c)A\underline{(a+c)} = (c-b)B\underline{(c+b)}$

Since $A \neq 0$; $B \neq 0$

<u>either</u> $a-c = c-b = 0 \Rightarrow a=b=c$

<u>OR</u>: $a+c = b+c \Rightarrow a=b$

but then $aA + aB = cC = c(A+B)$

and $a = c \; (=b)$