# PHYS 100C, Homework #4, Due Thursday, May 6th, 8AM (in class)

**Problem 10.13** A particle of charge $q$ moves in a circle of radius $a$ at constant angular velocity $\omega$. (Assume that the circle lies in the $xy$ plane, centered at the origin, and at time $t = 0$ the charge is at $(a, 0)$, on the positive $x$ axis.) Find the Liénard-Wiechert potentials for points on the $z$ axis.

**Problem 10.17** Derive Eq. 10.63. First show that

$$\frac{\partial t_r}{\partial t} = \frac{\mathscr{r} c}{\boldsymbol{\mathscr{r}} \cdot \mathbf{u}}.$$

**Problem 10.20** For the configuration in Prob. 10.13, find the electric and magnetic fields at the center. From your formula for **B**, determine the magnetic field at the center of a circular loop carrying a steady current $I$, and compare your answer with the result of Ex. 5.6

**Problem 10.21** Suppose you take a plastic ring of radius $a$ and glue charge on it, so that the line charge density is $\lambda_0 |\sin(\theta/2)|$. Then you spin the loop about its axis at an angular velocity $\omega$. Find the (exact) scalar and vector potentials at the center of the ring. [*Answer:* $\mathbf{A} = (\mu_0 \lambda_0 \omega a / 3\pi) \left\{ \sin[\omega(t - a/c)]\, \hat{\mathbf{x}} - \cos[\omega(t - a/c)]\, \hat{\mathbf{y}} \right\}$]

**Problem 10.25** A particle of charge $q$ is traveling at constant speed $v$ along the $x$ axis. Calculate the total power passing through the plane $x = a$, at the moment the particle itself is at the origin. [*Answer:* $q^2 v / 32\pi\epsilon_0 a^2$]

**Problem 11.1** Check that the retarded potentials of an oscillating dipole (Eqs. 11.12 and 11.17) satisfy the Lorentz gauge condition. Do *not* use approximation 3.

# PHYS-100C, Homework #4 Solutions

Thursday, April 30, 2009
2:30 PM

**10.1** $L = \nabla\cdot A + \mu_0\varepsilon_0 \cdot \frac{\partial V}{\partial t}$

$$\Box^2 V + \frac{\partial L}{\partial t} = \nabla^2 V - \cancel{\mu_0\varepsilon_0 \frac{\partial^2 V}{\partial t^2}} + \frac{\partial}{\partial t}(\nabla\cdot A) +$$

$$+ \cancel{\mu_0\varepsilon_0 \frac{\partial^2 V}{\partial t^2}} = \nabla^2 V + \frac{\partial}{\partial t}(\nabla\cdot A) = -\frac{\rho}{\varepsilon_0}$$

$$\Box^2 A - \nabla L = \nabla^2 A - \mu_0\varepsilon_0 \frac{\partial^2 A}{\partial t^2} - \nabla\left(\nabla\cdot A + \mu_0\varepsilon_0 \frac{\partial V}{\partial t}\right) =$$

$$= -\mu_0 \vec{J}$$

**10.3** $V = 0$

$A = -\frac{1}{4\pi\varepsilon_0} \cdot \frac{qt}{r^2}\hat{r}$

$$E = -\underbrace{\nabla V}_{0} - \frac{\partial A}{\partial t} = \frac{q}{4\pi\varepsilon_0} \cdot \frac{\hat{r}}{r^2}$$

Field from stationary point charge, at $r = 0$

$\rho = q \cdot \delta^3(r)$, $\vec{J} = 0$

$B = \nabla\times A = 0$

**10.5** $V' = V - \frac{\partial\lambda}{\partial t} = \frac{q}{4\pi\varepsilon_0 r}$

$$A' = A + \nabla\lambda = -\frac{qt}{4\pi\varepsilon_0 r^2}\hat{r} + \left(-\frac{qt}{4\pi\varepsilon_0}\right)\left(-\frac{\hat{r}}{r^2}\right) =$$

$$= 0$$

**10.7** $\Phi = \nabla \cdot A + \mu_0 \epsilon_0 \frac{\partial V}{\partial t}$

How does $\Phi$ change under gauge transformation?

$$\Phi' = \nabla \cdot A' + \frac{1}{c^2} \cdot \frac{\partial V'}{\partial t} = \nabla \cdot A + \nabla^2 \lambda + \frac{1}{c^2}\frac{\partial V}{\partial t} -$$

$$- \frac{1}{c^2}\frac{\partial^2 \lambda}{\partial t^2} = \Phi + \underbrace{\left(\nabla^2 - \frac{1}{c^2} \cdot \frac{\partial^2}{\partial t^2}\right)}_{}\lambda$$

"Square squared" $\rightarrow \Box^2$
as some of you call it

FOR ANY Given (scalar) function $\rho$ (charge distribution) we know exists a potential field $V$, such that:

$$\Box^2 V = -\frac{\rho}{\epsilon_0}$$

IF we replaced $\frac{\rho}{\epsilon_0}$ by $\Phi$, another random scalar field, there exists $\lambda^*$ such that

$$\Box^2 \lambda^* = -\Phi$$

Using that $\lambda^*$ for gauge tr.:

$$\Phi' = \Phi + \Box^2 \lambda^* = \Phi - \Phi = 0$$

for all $(\vec{r}, t)$.

We can always make $V \stackrel{!}{=} 0$

$$\frac{\partial \lambda}{\partial t} = V \Rightarrow \lambda = \int V \cdot dt' + \text{const}$$

Cannot generally make $A = 0 \Rightarrow$
$B = \nabla \times A = 0$

**10.10** $$\vec{A} = \frac{\mu_0}{4\pi} \int \frac{\vec{J} \cdot d\tau'}{\imath}$$

Since $d\tau' = \underset{\uparrow}{S} \cdot \underset{\uparrow}{dl}$
area length

$$\vec{J} \cdot d\tau' = \vec{J} \cdot S \cdot dl = I \cdot d\vec{l}$$

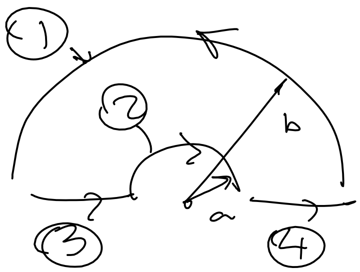
$$\vec{A} = \frac{\mu_0}{4\pi} \oint \frac{I(t_R) \cdot d\vec{l}}{\imath} =$$

$$= \frac{\mu_0}{4\pi} \oint \frac{k\left(t - \imath/c\right)}{\imath} d\vec{l} =$$

$$= \frac{\mu_0}{4\pi} \left( \oint \frac{kt}{\imath} \cdot d\vec{l} - \cancel{\oint \frac{k}{c} \cdot d\vec{l}} \right)$$
$0$

Since $\oint d\vec{l} = 0$
for any closed loop

$$\vec{A} = \frac{\mu_0}{4\pi} kt \oint \frac{d\vec{l}}{r}$$




(1): $\int_{1} \frac{d\vec{l}}{b} = -\frac{2b}{b} = -2$

(2): $\int_{2} \frac{d\vec{l}}{a} = +\frac{2a}{a} = 2$

(3): $\int_{-b}^{-a} \frac{dr}{|r|} = \ln\frac{b}{a}$

(4) $\int_{a}^{b} \frac{dz}{|z|} = \ln\frac{b}{a}$

(1) and (2) cancel, (3) & (4) the same:

$$\vec{A} = \frac{\mu_0}{4\pi} kt\left(-2+2+\ln\frac{b}{a}+\ln\frac{b}{a}\right)\hat{x} =$$

$$= \frac{\mu_0 kt}{2\pi}\ln\left(\frac{b}{a}\right)\cdot\hat{x}$$

$$\vec{E} = -\frac{\partial\vec{A}}{\partial t} = -\frac{\mu_0 k}{2\pi}\ln\left(\frac{b}{a}\right)\cdot\hat{x}$$

B is not defined since we can find A only at the center.