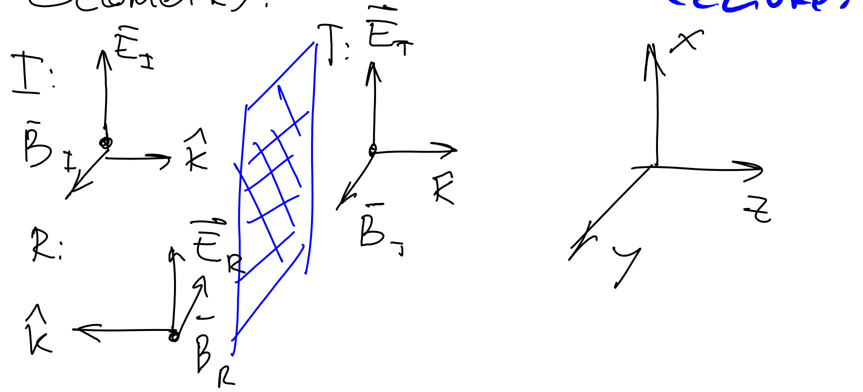
PHYS 100C, Lecture #5

Wednesday, April 07, 2010

Repeated from last lecture:

* Geometry:



* Reflected Wave: (R)

$$\vec{E}_R(z,t) = E_{0R}\, e^{i(-k_1 z - \omega t)}\,\hat{x}$$

$$\vec{B}_R(z,t) = -\frac{E_{0R}}{v_1}\, e^{i(-k_1 z - \omega t)}\,\hat{y}$$

Note "–" sign for k_1 in exp. as well as for $\vec{B}_R$ direction.

* Transmitted Wave: (T)

$$\vec{E}_T(z,t) = E_{0T}\, e^{i(k_2 z - \omega t)}\,\hat{x}$$

$$\vec{B}_T(z,t) = \frac{E_{0T}}{v_2}\, e^{i(k_2 z - \omega t)}\,\hat{y}$$

$$\omega = k_1 v_1 = k_2 v_2 = \text{const across boundary}$$

Boundary conditions for $z=0$

$$E_{0I} + E_{0R} = E_{0T} \qquad (i)$$

$$\frac{1}{\mu_1}\left(\frac{1}{v_1}\cdot E_{0I} - \frac{1}{v_1}E_{0R}\right) = \frac{1}{\mu_2}\left(\frac{1}{v_2}E_{0T}\right) \qquad (ii)$$

$$\frac{1}{\mu_1}\left(\frac{1}{v_1}\cdot E_{0I} - \frac{1}{v_1} E_{0R}\right) = \frac{1}{\mu_2}\left(\frac{1}{v_2} E_{0T}\right) \quad \text{(ii)}$$

(iii) AND (iv) are NOT USEFUL
SINCE $E^{\parallel} = B^{\parallel} = 0$ (NORMAL INCIDENCE)

(ii) REWRITTEN:

$$E_{0I} - E_{0R} = \frac{\mu_1 v_1}{\mu_2 v_2} E_{0T} \quad \text{(ii)}$$

*NOTE: IF WE MULTIPLY (i) by (ii):

$$E_{0I}^2 - E_{0R}^2 = \frac{\mu_1 v_1}{\mu_2 v_2} E_{0T}^2 \quad \text{, OR:}$$

$$\frac{1}{\mu_1 v_1} E_{0I}^2 = \frac{1}{\mu_1 v_1} E_{0R}^2 + \frac{1}{\mu_2 v_2} E_{0T}^2$$

$$\underset{\substack{\uparrow \\ \text{Incident Intensity}}}{\frac{1}{2}\varepsilon_1 v_1 E_{0I}^2} = \underset{\substack{\uparrow \\ \text{REFLECTED}}}{\frac{1}{2}\varepsilon_1 v_1 E_{0R}^2} + \underset{\substack{\uparrow \\ \text{TRANSM.}}}{\frac{1}{2}\varepsilon_2 v_2 E_{0T}^2}$$

ENERGY IS CONSERVED

* IF WE ADD (i)+(ii);
and use $\beta = \frac{\mu_1 v_1}{\mu_2 v_2} = \frac{\mu_1 n_2}{\mu_2 n_1}$

$$E_{0T} = \frac{2}{1+\beta} E_{0I}$$

$$E_{0R} = \frac{1-\beta}{1+\beta} E_{0I}$$

If: $\beta < 1$ $\quad E_{0R} > 0$,
REFL. is IN PHASE w/ Incident
$\beta > 1$ $\quad E_{0R} < 0$

Refl. wave is 180° out of phase

Since $\mu \sim \mu_0$ $\quad \beta \approx \frac{v_1}{v_2} = \frac{n_2}{n_1}$

$$\tilde{E}_{0T} \cong \left(\frac{2n_1}{n_1+n_2}\right) \tilde{E}_{0I}$$

$$\tilde{E}_{0R} \cong \left(\frac{n_1-n_2}{n_1+n_2}\right) \tilde{E}_{0I}$$

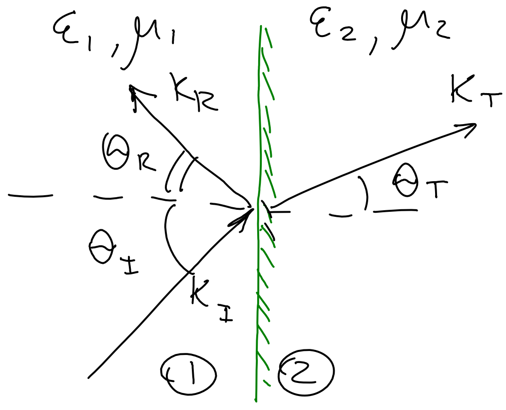
Reflection coefficient:

$$R \equiv \frac{I_R}{I_I} = \left(\frac{E_{0R}}{E_{0I}}\right)^2 = \left(\frac{n_1-n_2}{n_1+n_2}\right)^2$$

$$T \equiv \frac{I_T}{I_I} = \left(\frac{E_{0T}}{E_{0I}}\right)^2 \cdot \frac{\varepsilon_2 v_2}{\varepsilon_1 v_1} \simeq \frac{4 n_1 n_2}{(n_1+n_2)^2}$$

* Note that $R+T=1$ (as it should!)

* Oblique Incidence case:



Incident wave:

$$\tilde{\vec{E}}_I(\vec{r},t) = \tilde{E}_{0I}\, e^{i(\vec{k}_I\cdot\vec{r} - \omega_I t)}; \quad \vec{B}_I = \frac{(\vec{k}_I \times \vec{E}_I)}{v_1}$$

v_1

Reflected:

$$\tilde{E}_R(\vec{r},t) = \tilde{E}_{0R}\, e^{i(\vec{k}_R\cdot\vec{r}-\omega_R t)}; \quad B_R = \frac{(\vec{k}_R \times \vec{E}_R)}{v_1}$$

Transmitted:

$$\tilde{E}_T(\vec{r},t) = \tilde{E}_{0T}\cdot e^{i(\vec{k}_T\cdot\vec{r}-\omega_T t)}; \quad B_T = \frac{(\vec{k}_T \times \vec{E}_T)}{v_2}$$

where $k_I = \frac{\omega_I}{v_1}$; etc.

Boundary conditions: $(z=0)$

$$(\;)\cdot e^{i(\vec{k}_I\vec{r}-\omega_I t)} + (\;)\cdot e^{i(\vec{k}_R\vec{r}-\omega_R t)} = (\;)\cdot e^{i(\vec{k}_T\vec{r}-\omega_T t)}$$

As Prob. 9.15 shows, this means:

$\omega_I = \omega_R = \omega_T$ (frequencies are the same)

(We will use ω)

and

$$\vec{k}_I\cdot\vec{r} = \vec{k}_R\cdot\vec{r} = \vec{k}_T\cdot\vec{r}$$

$$(k_I)_x\cdot x + (k_I)_y\cdot y = (k_R)_x\cdot x + (k_R)_y\cdot y = (k_T)_x\cdot x + (k_T)_y\cdot y$$

($z=0$ so it doesn't enter)

For $x=0$:

$$(k_I)_y = (k_R)_y = (k_T)_y$$

For $y=0$

$$(k_I)_x = (k_R)_x = (k_T)_x$$

Let's define $\hat{x}$ axis so that k_I is in xz plane (of incidence)

or $(k_I)_y = 0$ $\left(= (k_R)_y = (k_T)_y\right)$

1st Law: $\vec{k}_I, \vec{k}_R, \vec{k}_T$ form a "plane of incidence", which also includes SURFACE normal

For x-components:

$$k_I \cdot \sin\theta_I = k_R \cdot \sin\theta_R = k_T \cdot \sin\theta_T$$

Since $k_I = k_R = \frac{\omega}{v_1}$ $\quad \theta_I = \theta_R$

2nd Law: ANGLE OF incidence is equal to the angle of reflection

$$\theta_I = \theta_R$$

(LAW OF REFLECTION)

FOR THIRD COMPONENT, $k_T = \frac{\omega}{v_2}$ and $k_I = \frac{\omega}{v_1}$:

$$\frac{\omega}{v_1} \cdot \sin\theta_I = \frac{\omega}{v_2} \cdot \sin\theta_T$$

$$\frac{\sin\theta_T}{\sin\theta_I} = \frac{v_2}{v_1} = \frac{n_1}{n_2}$$

3rd Law: Snell's Law or Law OF REFRACTION:

$$\frac{\sin\theta_T}{\sin\theta_I} = \frac{n_1}{n_2}$$