PHYS 100C, LECTURE 20

Friday, May 14, 2010
2:01 PM

$$\vec{E}(r,t) = \frac{q}{4\pi\epsilon_0} \cdot \frac{\eta}{(\eta \cdot u)^3} \left[\underbrace{(c^2 - v^2)\, u}_{\text{velocity field}} + \underbrace{\eta \times (u \times a)}_{\text{acc. field}} \right]$$

$$\vec{B} = \frac{\hat{\eta} \times \vec{E}}{c}$$

$$S = \frac{1}{\mu_0}(\vec{E} \times \vec{B}) = \frac{\vec{E}^2 \hat{\eta} - (\hat{\eta} \cdot \vec{E})\vec{E}}{\mu_0 c}$$

once again, E has

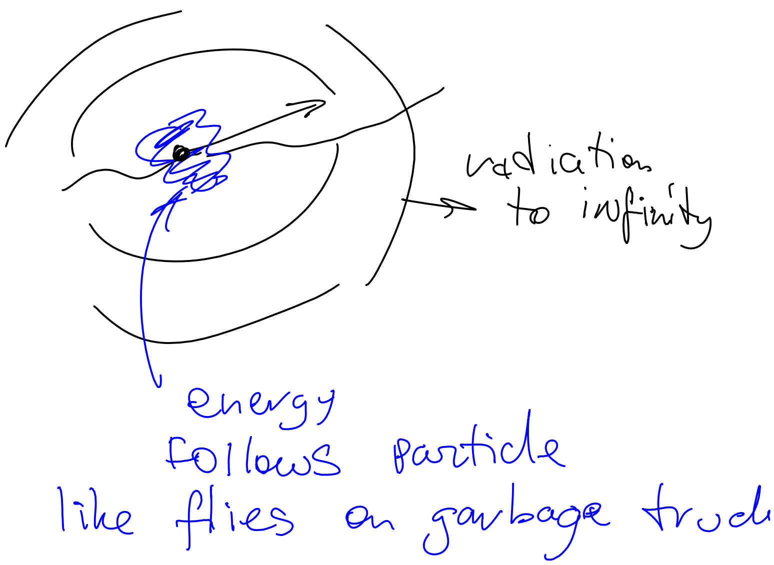
two terms; velocity field $\sim 1/\eta^2$
acceler. field $\sim 1/\eta$

For S:

accel. term^2 $\rightarrow \frac{1}{\eta^2}$ } radiation

velocity term^2 $\rightarrow \frac{1}{\eta^4}$ } "cloud" of energy

cross term $\rightarrow \frac{1}{\eta^3}$



Reaction force?

Larmor Law:

$$P = \frac{\mu_0 q^2 a^2}{6\pi c}$$

($\langle S \rangle$ integrated over $4\pi R^2$)

$$F \cdot v = -P = -\frac{\mu_0 q^2 a^2}{6\pi c}$$

(Power is force x velocity)

True only on average!
Wrong for instanteneous F.

Why? Neglects energy exchange with "entourage" energy cloud of garbage flies.

If we consider times t_1 and t_2 that have the same a and v, then $S(t_1) = S(t_2)$ and "cloud" energy is the same, so:

$$\int_{t_1}^{t_2} F \cdot v \cdot dt = -\frac{\mu_0 q^2}{6\pi c}\int_{t_1}^{t_2} a^2 dt$$

(reaction force due to energy radiated out).

but $a^2 = \left(\frac{\partial v}{\partial t}\right) \cdot \left(\frac{\partial v}{\partial t}\right) = \left(v \cdot \frac{\partial v}{\partial t}\right)' - v\frac{\partial^2 v}{\partial t^2}$

$$\int_{t_1}^{t_2} a^2 dt = v\frac{\partial v}{\partial t}\Bigg|_{t_1}^{t_2} - \int_{t_1}^{t_2} v\frac{\partial^2 v}{\partial t^2}$$

$= 0$ since $v(t_2) = v(t_1)$, $a(t_2) = a(t_1)$

$$\int_{t_1}^{t_2}\left(F\cdot v - \frac{\mu_0 q^2}{6\pi c}\dot{a}v\right)\cdot dt = 0$$

which is true if

$$\vec{F} = \frac{\mu_0 q^2}{6\pi c}\dot{\vec{a}}$$

Jerk → (just like Steve Martin movie)

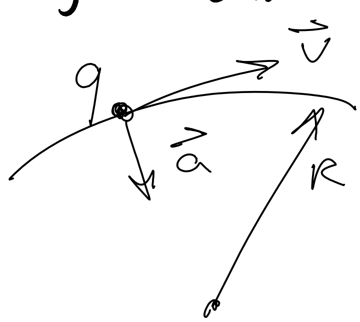
What does it mean?

Energy exchange w/ cloud is such that $\vec{F} \sim \dot{\vec{a}}$

E.g. if $a=0$ (no radiation) but $\dot{a}\neq 0$, there's force (creating or destroying "cloud").

(Weird stuff! (see textbook for more weirdness).

* Synchrotrons



Particle q moving on a circle R

Classical $a = \frac{v^2}{R}$

Relativistic:

$$a = \frac{1}{m}\cdot\frac{\partial p}{\partial \tau} = \frac{1}{m}\frac{\partial(\gamma m v)}{\partial t/\gamma} = \gamma^2\frac{\partial v}{\partial t} = \gamma^2\frac{v^2}{R}$$

$d\tau = \frac{dt}{\gamma}$ ← time contraction

$P \sim a^2$ (power radiated)

$P \sim \gamma^4 \frac{v^4}{R^2}$

Advanced Photon Source, Chicago IL

Energy of electrons 7 GeV
rest mass of e^- 0.5 MeV

$\gamma = \frac{7\,GeV}{0.5\,MeV} = 14{,}000$ (!)

Radiation is a problem for high energy physics.

LHC aims to ramp up from

450 GeV to 7 TeV
(they use protons, not electrons)

Increase energy by x10 →
power bill goes up by x10,000!

You can reduce it by making

R large (LHC is 27 km circumference).

Can we make it 2700 km
to cut power bill by x10,000?
$\left(P \sim \frac{1}{R^2}\right)$