PHYS 100C, Midterm exam. Wed., May 5th 10:00AM-10:50AM

1. a) From energy conservation considerations, find the power-law scaling dependence of time-average E and B fields of a spherically symmetrical wave in vacuum, on a distance R from the source.

Energy flux $\vec{S} = \bar{E} \times \bar{B} \sim \frac{1}{R^2}$ (since $\int \langle S \rangle \cdot 4\pi R^2 = \text{const}$)

since $|B| = \frac{|E|}{c}$, $|E|, |B| \sim \frac{1}{R}$

b) Write a general expression for E (x,y,z,t) and B (x,y,z,t) of a planar monochromatic wave with wavelength λ and amplitude E_0 polarized along (0, 1, 1) direction in y-z plane and propagating (in vacuum) along x direction.

$$E = \frac{E_0}{\sqrt{2}}(\hat{y} + \hat{z}) \cdot \cos\left(\frac{2\pi x}{\lambda} - \frac{2\pi c}{\lambda} t + \varphi\right)$$

$$B = \frac{E_0}{c\sqrt{2}}(\hat{z} - \hat{y}) \cdot \cos\left(\frac{2\pi x}{\lambda} - \frac{2\pi c}{\lambda} t + \varphi\right)$$

2. a) For shallow gravity water waves group velocity is

$$v_{group} = \sqrt{\frac{g\lambda}{2\pi}}$$

where λ is wavelength and g is a gravitational field strength. Find the ratio of phase and group velocities, $\frac{v_{phase}}{v_{group}}$.

$$\frac{\partial \omega}{\partial k} = \sqrt{\frac{g\lambda}{2\pi}} = \sqrt{\frac{g}{k}}$$

$$\omega = \int \sqrt{\frac{g}{k}} \cdot dk = 2\sqrt{gk} \ (+\text{const})$$

$$V_{phase} = \frac{\omega}{k} = 2\sqrt{\frac{g}{k}}$$

$$\frac{V_{group}}{V_{phase}} = \frac{1}{2}$$

b) Do the same for capillary waves, for which

$$v_{group} = \sqrt{\frac{2\pi\sigma}{\rho\lambda}}$$

where σ is the surface tension and ρ is the liquid density (both are constants, naturally).

$$\frac{\partial \omega}{\partial k} = \sqrt{\frac{\sigma k}{\rho}} \qquad \omega = \int \sqrt{\frac{\sigma k}{\rho}} \cdot dk = \sqrt{\frac{\sigma}{\rho}} \cdot \frac{2}{3} k^{3/2}$$

$$V_{phase} = \frac{\omega}{k} = \frac{2}{3}\sqrt{\frac{\sigma}{\rho}} \cdot k^{1/2} \qquad \frac{V_{group}}{V_{phase}} = \frac{3}{2}$$

3. Rectangular waveguide has dimensions a=0.25 mm and b=0.20 mm.

a) List all TE_{mn} modes excited by EM wave with wavelength λ=0.21 mm.

b) What range of wavelengths will excite three and only three modes?

(Hint: if you are clever enough, you will not need the value for speed of light, c, in these calculations)

Allowed modes TE_{mn}

$$\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 < \left(\frac{2}{\lambda}\right)^2 \qquad \frac{2}{\lambda} = 9.52\ mm^{-1}$$

m	n	$\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2}$, mm^{-1}	
0	0	NOT ALLOWED	
1	0	4	yes
0	1	5	yes
1	1	6.4	yes
2	0	8	yes
2	1	9.43	yes
0	2	10	NO
3	0	12	NO

A) Answer:

TE_{10}, TE_{01}, TE_{11}, TE_{20} and TE_{21}

B) 3rd lowest mode is TE_{11}

$$6.4\ mm^{-1} < \frac{2}{\lambda} < 8\ mm^{-1}$$

$$0.25\ mm < \lambda < 0.312\ mm$$