PHYSICS 100C Final Exam, Thursday, June 7, 8AM-11AM

1. A loop of radius R has "glued" linear charge density distribution as a function of azimuthal angle φ, $\rho(\varphi)=\rho_0 \cos^2\varphi$. The loop is spinning with angular frequency ω. Find retarder potentials (in Lorentz gauge) A and V at the center of the loop.

2. Show that the x-rays (for which frequency much greater than resonance frequencies typically in UV range $\omega >> \omega_j$) have an index of refraction less than 1, and estimate the angle of total internal reflection for 10 KeV x-rays incident on vacuum/metal interface at a grazing incidence. You may assume $N=10^{30}$ "free" electrons per m^3 in a metal.

3. **a)** A loop of radius R (yes, yet another spinning charged loop problem!) is centered at (0,0,0) and is oriented in x-y plane as shown on the Fig. 1. Upper half of the loop has "glued" uniform linear charge density +ρ, and bottom half of the loop has linear charge density -ρ.

Find total power radiated if the loop is spinning around its axis (with respect to z axis) at angular frequency ω.

b) How would the answer change if the loop is instead spinning around x axis (at the same frequency)? How about if it was spinning around y-axis?

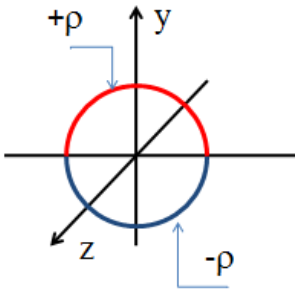

Figure 1 (problem 3)

4. **a)** Electric and magnetic fields at some region of space are given by two vectors, **E** and **B**, such as **E** ⊥ **B.** Show that there always exists a frame of reference in which **B**=0, and find its velocity (direction and magnitude), as well as the resulting electric field **E'** in this frame of reference.

b) Show that for two arbitrary (no longer mutually perpendicular) vectors **E** and **B**, their scalar product **E**·**B** is relativistically-invariant. Comment on existence of frame of reference found in a) when **E** and **B** are not perpendicular.

5. Large Hadron Collider is set to collide two beams of protons travelling at the same speed in opposite directions with energy of 7 TeV per proton (in the laboratory frame). What is the energy of on-coming protons in the frame of reference of one of the beams? (Take rest mass of a proton to be 1 GeV).