PHYS 100C, Lecture 6

Friday, April 09, 2010
10:00 AM

* **Boundary conditions:**
  (exponents cancel out)

(i) $\varepsilon_1(\tilde{E}_{0I}+\tilde{E}_{0R})_z=\varepsilon_2(\tilde{E}_{0T})_z$

(ii) $(\tilde{B}_{0I}+\tilde{B}_{0R})_z=(\tilde{B}_{0T})_z$

(iii) $(\tilde{E}_{0I}+\tilde{E}_{0R})_{x,y}=(\tilde{E}_{0T})_{x,y}$

(iv) $\frac{1}{\mu_1}(\tilde{B}_{0I}+\tilde{B}_{0R})_{x,y}=\frac{1}{\mu_2}(\tilde{B}_{0T})_{x,y}$

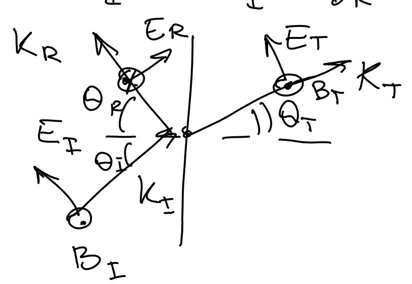
and $\bar{B}=\frac{1}{v}(\bar{k}\times\bar{E})$

For p-polalized wave
($\bar{E}$ || plane of incidence):

(i)$\Rightarrow$ $\varepsilon_1(-\tilde{E}_{0I}\cdot\sin\theta_I+\tilde{E}_{0R}\cdot\sin\theta_R)=-\varepsilon_2 E_{0T}\sin\theta_T$

$B\perp z$, so (ii): $0=0$

(iii) $\tilde{E}_{0I}\cdot\cos\theta_I+\tilde{E}_{0R}\cdot\cos\theta_R=\tilde{E}_{0T}\cdot\cos\theta_T$



(iv)$\Rightarrow$ $\frac{1}{\mu_1 v_1}(\tilde{E}_{0I}-\tilde{E}_{0R})=\frac{1}{\mu_2 v_2}\tilde{E}_{0T}$

since $\theta_I=\theta_R$, (i) & (iv):

$$\tilde{E}_{0I}-\tilde{E}_{0R}=\tilde{E}_{0T}\cdot\beta$$

where $\beta \equiv \frac{\mu_1 v_1}{\mu_2 v_2} = \frac{\varepsilon_2}{\varepsilon_1} \cdot \frac{\sin\theta_T}{\sin\theta_I} = \frac{\mu_1 n_2}{\mu_2 n_1}$

(iii) $\Rightarrow$ $\tilde{E}_{0I} + \tilde{E}_{0R} = \tilde{E}_{0T} \cdot \alpha$

$$\alpha \equiv \frac{\cos\theta_T}{\cos\theta_I}$$

add / subtract: **Fresnel Eqs:**

$$\boxed{\tilde{E}_{0R} = \left(\frac{\alpha - \beta}{\alpha + \beta}\right)\tilde{E}_{0I} \; ; \; \tilde{E}_{0T} = \left(\frac{2}{\alpha+\beta}\right)\tilde{E}_{0I}}$$

since $\cos\theta_T = \sqrt{1 - \sin^2\theta_T} = \sqrt{1 - \frac{n_1^2}{n_2^2} \cdot \sin^2\theta_I}$

$$\alpha(\theta_I) = \sqrt{1 - \frac{n_1^2 \sin^2\theta_I}{n_2^2}} \cdot \frac{1}{\cos\theta_I}$$

$$R \equiv \frac{I_R}{I_I} = \left(\frac{E_{0R}}{E_{0I}}\right)^2 = \left(\frac{\alpha - \beta}{\alpha + \beta}\right)^2$$

$$T \equiv \frac{I_T}{I_I} = \frac{\varepsilon_2 v_2}{\varepsilon_1 v_1} \cdot \left(\frac{E_{0T}}{E_{0I}}\right)^2 \frac{\cos\theta_T}{\cos\theta_I} = \alpha\beta\left(\frac{2}{\alpha+\beta}\right)^2$$

* Brewster Angle:

when $\alpha = \beta$ $\quad R = 0$

$\alpha = \beta \Rightarrow \quad 1 - \left(\frac{n_1}{n_2}\right)^2 \cdot \sin^2\theta_B = \beta^2 \cdot \cos^2\theta_B = \beta^2(1 - \sin^2\theta_B)$

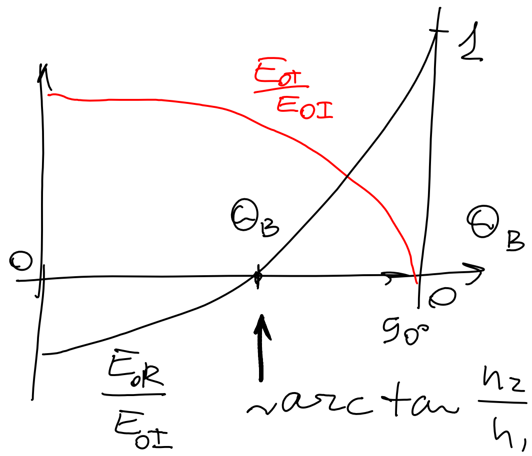
$$\sin^2\theta_B = \frac{1 - \beta^2}{(n_1/n_2)^2 - \beta^2} \qquad \text{since } \beta \approx \frac{n_2}{n_1}$$

$$\sin^2\theta_B = \frac{1 - \beta^2}{\left(\frac{1}{\beta}\right)^2 - \beta^2} = \beta^2 \frac{1 - \beta^2}{1 - \beta^4} = \frac{\beta^2}{1 + \beta^2}$$

OR

$$\tan^2\theta_B = \frac{\sin^2\theta_B}{1 - \sin^2\theta_B} = \beta^2$$

$$\tan\theta_B = \frac{n_2}{n_1}$$



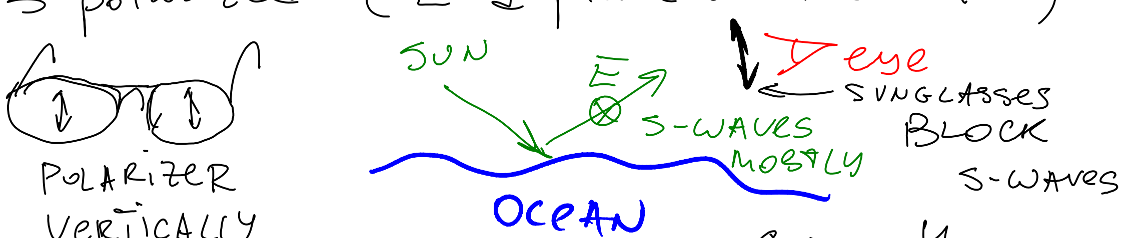
E_{0R} is < 0 (out of phase by $180°$ w.r. to E_{0I})

* Application #1: polarized SUNGLASSES

p-polarized waves ⇒ reflection is suppresed around θ_B ($R = 0$ at θ_B)

s-polarized waves "survive" reflection

Result: reflected sunlight is preferentially s-polarized ($E \perp$ plane of incidence)



CUTS the "SUN GLARE"

* APPLICATION #2
Brewster ANGLE MICROSCOPY: