PHYS-100C, Homework #4, due Thursday, April 30.

Problem 10.1 Show that the differential equations for V and $\mathbf{A}$ (Eqs. 10.4 and 10.5) can be written in the more symmetrical form

$$\left.\begin{aligned} \Box^2 V + \frac{\partial L}{\partial t} &= -\frac{1}{\epsilon_0}\rho, \\ \Box^2 \mathbf{A} - \nabla L &= -\mu_0 \mathbf{J}. \end{aligned}\right\} \qquad (10.6)$$

where

$$\Box^2 \equiv \nabla^2 - \mu_0\epsilon_0 \frac{\partial^2}{\partial t^2} \quad \text{and} \quad L \equiv \nabla \cdot \mathbf{A} + \mu_0\epsilon_0 \frac{\partial V}{\partial t}.$$

Problem 10.3 Find the fields, and the charge and current distributions, corresponding to

$$V(\mathbf{r}, t) = 0, \quad \mathbf{A}(\mathbf{r}, t) = -\frac{1}{4\pi\epsilon_0}\frac{qt}{r^2}\hat{\mathbf{r}}.$$

Problem 10.5 Use the gauge function $\lambda = -(1/4\pi\epsilon_0)(qt/r)$ to transform the potentials in Prob. 10.3, and comment on the result.

Problem 10.7 In Chapter 5, I showed that it is always possible to pick a vector potential whose divergence is zero (Coulomb gauge). Show that it is always possible to choose $\nabla \cdot \mathbf{A} = -\mu_0\epsilon_0(\partial V/\partial t)$, as required for the Lorentz gauge, assuming you know how to solve equations of the form 10.16. Is it always possible to pick $V = 0$? How about $\mathbf{A} = 0$?

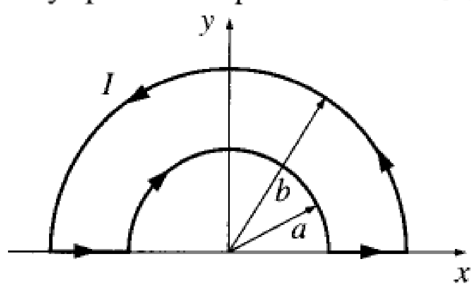

Figure 10.5

Problem 10.10 A piece of wire bent into a loop, as shown in Fig. 10.5, carries a current that increases linearly with time:

$$I(t) = kt.$$

Calculate the retarded vector potential $\mathbf{A}$ at the center. Find the electric field at the center. Why does this (neutral) wire produce an *electric* field? (Why can't you determine the *magnetic* field from this expression for $\mathbf{A}$?)