

PHYS 100C, Lecture #17

Monday, May 10, 2010
9:24 PM

RADIATION:

Where do waves (FROM CHAPTER 9)
come FROM? Accelerated charges.

Lienard-Wichert:

$$E \sim \underbrace{\frac{1}{(ru)^2}}_{\text{Scales as } \frac{1}{r^2} \text{ (Coulomb-like)}} + \underbrace{\frac{r(\ddot{r}_x(ua))}{(ru)^3}}_{\text{Scales as } \frac{1}{r} \sim a \text{ (acceleration)}}$$

Because $B = \frac{\hat{r} \times \vec{E}}{c}$ (LW potentials),
 $B \sim \frac{1}{r^2} + \frac{a}{r}$

Radiated intensity:

$$S \sim E \times B \sim \underbrace{\frac{a^2}{r^2}}_{\text{acceleration term}} + \underbrace{\frac{a}{r^3}}_{\text{cross terms}} + \underbrace{\frac{1}{r^4}}_{\text{Coulomb term}}$$

For LARGE distances, integrated over a sphere of radius r , the acceleration term $\sim a^2/r^2$ survives, since total power radiated is $\sim a^2/r^2 \cdot 4\pi r^2 \sim a^2 = \text{const.}$

Higher-order contributions decay,
For harmonic oscillator

$$x = x_0 \cos \omega t$$

$$x' = -x_0 \omega \sin \omega t$$

$$a = x'' = -\omega^2 x_0 \cos \omega t$$

Since $a \sim \omega^2$, radiated EM wave intensity $P \sim \langle S \rangle \sim \frac{a^2}{r^2} \sim \frac{\omega^4}{r^2}$

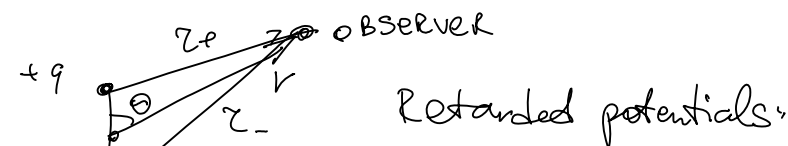
(HAND-WAVING ARGUMENT).

Here's more DETAILED derivation.

Consider dipole oscillating
at frequency ω :

$$d \begin{array}{c} +q_0 \\ -q_0 \end{array} \quad \vec{p}(t) = p_0 \cdot \cos(\omega t) \cdot \hat{z}$$

$p_0 = q_0 \cdot d$



$$V = \frac{q_0}{4\pi\epsilon_0} \left[\frac{\cos[\omega(t - \frac{r_+}{c})]}{r_+} - \frac{\cos[\omega(t - \frac{r_-}{c})]}{r_-} \right]$$

$$r_{\pm}^2 = r^2 \mp r d \cos\theta + \left(\frac{d}{2}\right)^2$$

$$\text{If } d \ll r \Rightarrow r_{\pm} = r \left(1 \mp \frac{d}{2r} \cos\theta \right)$$

$$\text{Also } \frac{1}{1 \pm x} \approx 1 \mp x \Rightarrow \frac{1}{r_{\pm}} = \frac{1}{r} \left(1 \pm \frac{d}{2r} \cos\theta \right)$$

$$p = \frac{\Delta r \cdot \omega}{c} = \frac{d \cdot \cos\theta \cdot \omega}{2c} = \frac{\pi d \cdot \cos\theta}{\lambda} \ll 1$$

$$\text{since } \frac{\omega}{c} = \frac{2\pi}{\lambda} \quad (\text{or } d \ll \lambda)$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B \approx 1 - \frac{p^2}{2}$$

$$\cos\left[\omega\left(t - \frac{r_{\pm}}{c}\right)\right] = \cos(\omega t - kr) \cdot \cos p_{\mp}$$

$$\mp \sin(\omega t - kr) \sin p \approx$$

$$\approx \cos\left(\omega t - \frac{r}{\lambda}\right) \mp p \cdot \sin(\omega t - kr)$$

(dropping terms $\sim p^2$)

Plugging $\frac{1}{z_{\pm}}$ and $\cos(\omega t - kz_{\pm})$ into expression for $V(r, t)$:

$$V = \frac{q_0}{4\pi\epsilon_0} \left(\frac{\cos(\omega t - kz) - \sin(\omega t - kz) \frac{\pi d \cos\theta}{\lambda}}{z} \left(1 + \frac{d \cos\theta}{2z}\right) - \frac{\cos(\omega t - kz) + \sin(\omega t - kz) \frac{\pi d \cos\theta}{\lambda}}{z} \left(1 - \frac{d \cos\theta}{2z}\right) \right) =$$

$$= \frac{q_0}{4\pi\epsilon_0 z} \left(\cancel{\cos(\omega t - kz)} - \sin(\omega t - kz) \cdot \frac{\pi d \cos\theta}{\lambda} + \cos(\omega t - kz) \frac{d \cos\theta}{2z} + \cancel{\cos(\omega t - kz)} - \frac{\pi d \cos\theta}{2z\lambda} - \cancel{\sin(\omega t - kz)} + \sin(\omega t - kz) \frac{\pi d \cos\theta}{\lambda} + \cos(\omega t - kz) \frac{d \cos\theta}{z} + \sin(\omega t - kz) \frac{\pi d^2 \cos^2\theta}{2z\lambda} \right)$$

We will neglect $\sim \frac{d^2}{z\lambda}$ term (2nd order)

Zeroth order term $\cos(\omega t - kz)$ cancels out

Leading terms are 1st order: $\sim \frac{d}{\lambda}$ (blue)

and $\sim \frac{d}{z}$ (green). Take d and $\cos\theta$ outside

and use $p_0 = q_0 \cdot d$: main term

$$V = \frac{p_0 \cos\theta}{4\pi\epsilon_0 z} \left[\frac{-2\pi \sin(\omega t - kz)}{\lambda} + \frac{\cos(\omega t - kz)}{z} \right]$$

Check: for $\omega \rightarrow 0$ $V = \frac{p_0 \cos\theta}{4\pi\epsilon_0 z^2}$

(static dipole potential)

Two terms: $\sim \frac{1}{\lambda}$ and $\sim \frac{1}{z}$

For large $r \gg \lambda$, keep only $\frac{1}{z}$ term

$$V = -\frac{p_0 \cos\theta}{4\pi\epsilon_0 z \lambda} \sin(\omega t - kz)$$

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Current in oscillating dipole:

$$I(t) = \frac{dq}{dt} \hat{z} = -q_0 \omega \sin \omega t \hat{z}$$

$$A = \frac{\mu_0}{4\pi} \int_{-d/2}^{d/2} \frac{-q_0 \omega \sin(\omega t - kz) \hat{z}}{r} dz$$

$$A = -\frac{\mu_0}{4\pi} \cdot \frac{p_0 \omega}{r} \sin(\omega t - kz) \hat{z}$$

$$\nabla V = \frac{\partial V}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial V}{\partial \theta} \hat{\theta}$$

$$\frac{\partial V}{\partial z} = \dots \frac{\sin(\dots)}{r^2} - \frac{\cos(\dots)}{r\lambda}$$

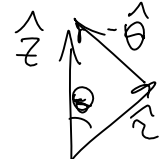
small ($r \gg \lambda$)

$$\frac{1}{r} \frac{\partial V}{\partial \theta} \sim \frac{1}{r^2} \quad (\text{also small compared to } \sim \frac{1}{r} \text{ term})$$

$$\nabla V = \frac{p_0}{4\pi\epsilon_0 \lambda^2} \cdot \frac{\cos \theta}{r} \cdot \cos(\omega t - kz) \hat{r}$$

Similarly

$$\frac{\partial A}{\partial t} = -\frac{\mu_0}{4\pi} \cdot \frac{p_0 \omega^2}{r} \cos(\omega t - kz) \hat{z}$$

$$\hat{z} = \hat{r} \cos \theta - \sin \theta \hat{\theta}$$


$$E = -\nabla V - \frac{\partial A}{\partial t} = -\frac{\mu_0}{4\pi} p_0 \omega^2 \left(\frac{\sin \theta}{r} \right) \cos(\omega t - kz) \hat{\theta}$$

(the $\cos\theta$ terms in ∇V and $\frac{\partial A}{\partial t}$ cancel)

$$\nabla \times A = \frac{1}{z} \left[\frac{\partial}{\partial z} (r A_\theta) - \frac{\partial A_z}{\partial \theta} \right] \hat{\phi}$$

$$B = \nabla \times A = -\frac{\mu_0 p_0 \omega^2}{4\pi c} \left(\frac{\sin\theta}{r} \right) \cos(\omega t - kz) \hat{\phi}$$

(or, could have used

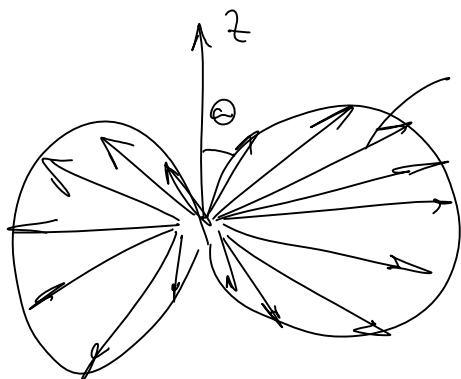
$$\vec{B} = \frac{\hat{z}}{c} \times \vec{E}, \text{ from } (\omega) \text{ potentials}$$

\vec{E}, \vec{B} transverse ($\perp \hat{z}, \vec{E} \perp \vec{B}$),
in phase, just like EM waves

$$\vec{S} = \frac{1}{\mu_0} [\vec{E} \times \vec{B}] = \frac{\mu_0}{c} \left(\frac{p_0 \omega^2}{4\pi} \left(\frac{\sin\theta}{r} \right) \cos(\omega t - kz) \right)^2 \hat{r}$$

$$\langle S \rangle = \frac{\mu_0 p_0^2 \omega^4}{32\pi^2 c} \cdot \frac{\sin^2\theta}{r^2} \cdot \hat{r}$$

(since $\langle \cos^2(\omega t - \frac{z}{\lambda}) \rangle = \frac{1}{2}$)



$\sin^2\theta$

No scattering
along dipole axis (\hat{z}).
"Donut" shape in 3D

$\langle S \rangle \sim \omega^4 \leftarrow$ Rayleigh

scattering, reason

why sky is blue, sunset is red, etc.